Robert A. Beeler
150 Ligon Street, #1002
Clemson, SC 29631
http://people.clemson.edu/∼rbeeler/mypage.html

Cell: (865) 603-1131
Home: (864) 653-8639
Work: (864) 654-5196
Email: rbeeler@clemson.edu

Statement of Research

My dissertation research has focused on problems relating to edge decompositions of graphs and the properties of the intersection graph generated by the decomposition. Given a family of graphs $\mathcal{K}$, an edge decomposition,$\mathscr{D}$, of a graph H (called the *host*) is a partition of the edge set of H such that each part of the partition is isomorphic to an element of $\mathcal{K}$. Traditionally, the parts of this partition are called *blocks* and the elements of $\mathcal{K}$ are called *prototypes* or *divisors*. Of particular interest is the case where $\mathcal{K}$ consists of only a single prototype, G, in which case we say that $\mathscr{D}$ is a G-decomposition of H. The intersection graph generated by such a decomposition, denoted $I(\mathscr{D})$ has a vertex for each block in the partition and two vertices share a common edge in $I(\mathscr{D})$ if and only if the corressponding blocks share a common vertex in H. If $I(\mathscr{D})$ is isomorphic to H, we say that $\mathscr{D}$ is an *automorphic* G-decomposition of H.

Most of the known examples of these decompositions are obtained by cyclic decompositions. Informally, a *cyclic* decomposition, is one is which there exists a cyclic permutation g, such that if $A, B \in \mathscr{D}$ are G-blocks of the the decomposition, then the block A may be obtained by applying g^i to B for some $i \in \mathbb{N}$. It is well known that a graceful labelling will induce a cyclic decomposition of a complete graph, and in fact this decomposition is automorphic. $f : V(G) \rightarrow \{0, ..., |E(G)|\}$ is a *graceful labelling* on G provided that f is injective and for all $uv \in E(G)$, $|f(u) - f(v)| \in \{1, ..., |E(G)|\}$ is unique. Thus my work is connected to several unsolved problems in graph theory, such as Ringel's Conjecture and the Graceful Tree Conjecture. While a graceful labelling is sufficient to induce an automorphic G-decomposition of a complete graph, other related labellings will also work. I have also investigated construction methods for these labellings as a means of obtaining an automorphic decomposition for certain generalizations of the complete graph.

I have also concentrated my research on the necessary conditions for the existence of an automorphic decomposition. These necessary conditions have been used to classify certain graphs which admit an automorphic decomposition. In particular, I have shown that:

- If H admits an automorphic P_2-decomposition, then H is the disjoint union of cycles;

- If $\chi(H) = n(G)$, G is d-regular, and H admits an automorphic G decomposition, then H is $n(G)d$-regular;
- If $\chi(H) = n(G)$, G is a p-star, and H admits an automorphic G decomposition, then H is p-regular;
- If G is d-regular and $\mathscr{D}$ is an automorphic G-decomposition of H such that any two G-blocks share at most one common vertex in H, then H is $n(G)d$-regular and G is a complete graph.

My future research plans include extending this characterization.

My research is related to several classic problems in graph theory and combinatorial designs. As previously mentioned, the labellings on graphs that will induce and automorphic decomposition are related to graceful labelings. It is also possible to obtain these decompositions through the existence of projective planes. As such, my work is related to the Prime Power Conjecture and the existence of perfect difference sets.

In addition to my specialization in edge decompositions and labellings on graphs, I am also interested in several areas of discrete mathematics. In addition to the combinatorial arguments used to prove certain graph theoretical results, I am interested in formal power series, as well as their application to generating functions. Several of the aforementioned labellings on graphs arise from groups. I have also used several classical results from number theory, such as Bertrand's Postulate and the Frobenius number to prove certain results. I am also interested in several areas of general mathemtical interest, such as the History and Philosophy of Mathematics. I have given several seminar presentations on these topics, which are listed in the enclosed vita. Also, you may find their abstracts at:

`http://people.clemson.edu/~rbeeler/seminar.pdf`

Through my studies, I have grown as a mathematician and as a teacher. Many of the proof techniques used in my research are difficult, relying on several different areas of mathematics. I find that this gives me a great sense of perspective to be working on something that is as difficult for me as calculus is for my students. It is important for students to understand that mathematics is a growing field, with discovery taking place through research. I also find that by researching the history of mathematics, I can give my students a better motivation for the material in class. It is also important to keep abreast of ongoing research in mathematical education, and incorporate new techniques into the classroom.