Mathematical Modeling of College Student Enrollment

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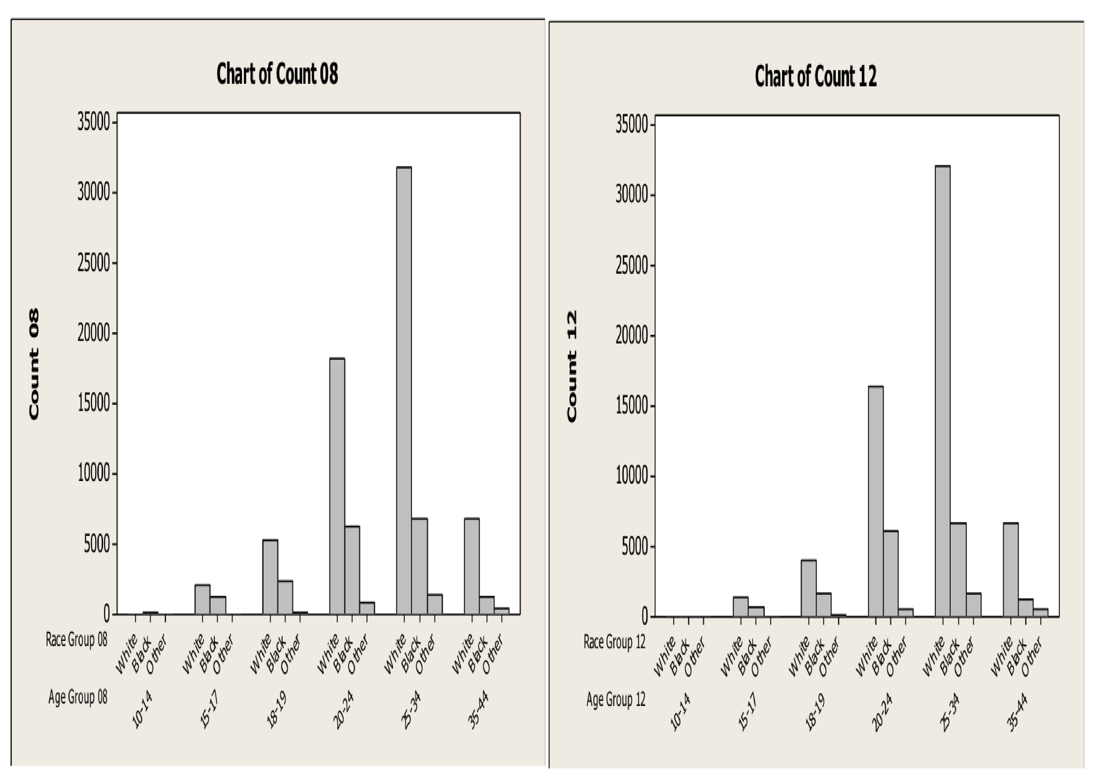
Objectives

- To develop a mathematical model similar to the Leslie matrix to predict enrollment.
- To analyze data from ETSU and determine what affects student enrollment.
- Use this information to improve our model to make more accurate predictions.
- Test this models with known data to determine their accuracy.

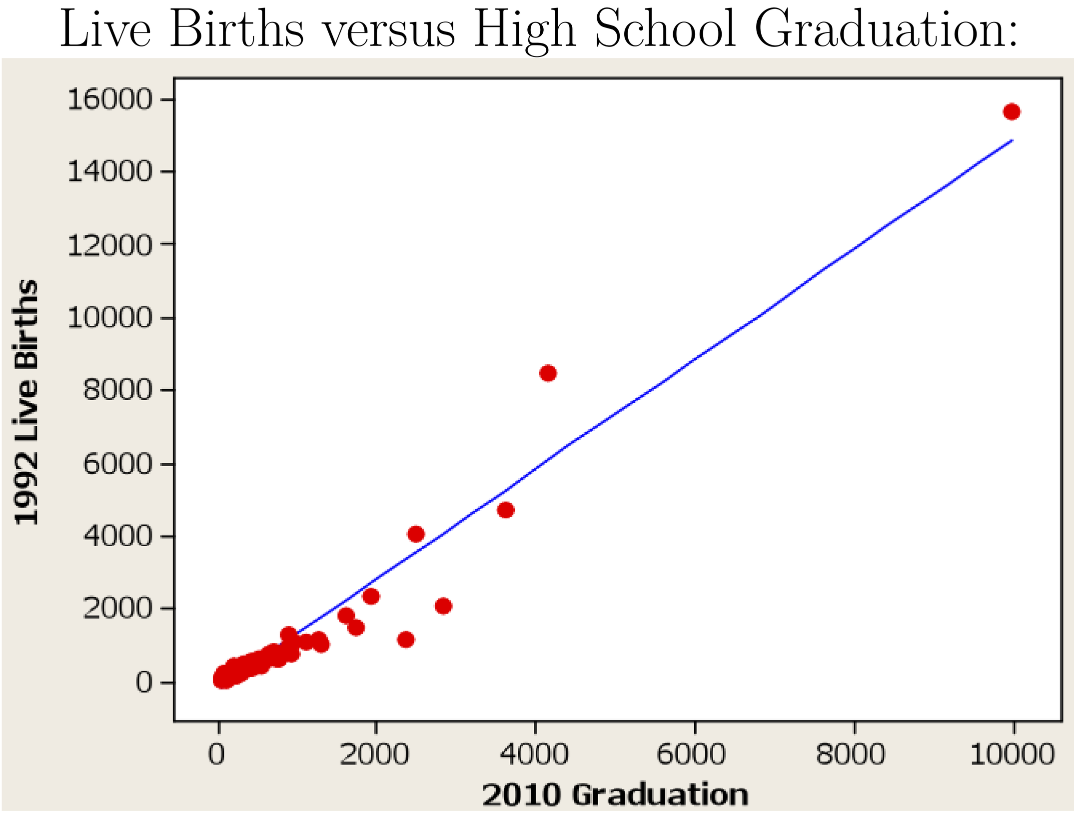
Analysis of Demographic Data

Number of Live Births in TN: 2008 vs. 2012



	Group 10-14 year-old	Group 15-17 year-old
2008	150	3328
2012	89	2117

Live Births versus High School Graduation:



Correlation = 0.9696589

$R^2 = 94.0\%$

$y = 1.507x - 215$

Leslie Model

$$x(n+1) = Ax(n)$$

$$x(n) = A^n x(0)$$

$$x(n) = \sum_{j=0}^{m} c_j(n) v_j = c_0(0)\lambda_0^n v_0 + u_n$$

$$\frac{x(n)}{P(n)} = \frac{c_0(0)v_0 + \lambda_0^{-n} u_n}{c_0(0)\sum_{j=0}^{m} v_0(j) + \lambda_0^{-n}\sum_{j=0}^{m} u_n(j)}$$

Stable Age Distribution:

$$\frac{x(n)}{P(n)} \to \frac{v_0}{\sum_{j=0}^{m} v_0(j)}$$

Characteristic Equation:

$$\lambda^{m+1} - \beta_0\pi_0\lambda^m - \beta_1\pi_1\lambda^{m-1} - \ldots - \beta_m\pi_m = 0$$

Modified Leslie Model

$$x(n+1) = Ax(n) + b$$

$$x(n) = A^n x(0) + (A^n - I)(A - I)^{-1} b$$

$$x(n+1) = Ax(n) + b = \sum_{j=0}^{m} c_j(n) A v_j + \sum_{j=0}^{m} \alpha_j v_j$$

$$x(n+1) = \sum_{j=0}^{m} c_j(n)\lambda_j v_j + \sum_{j=0}^{m} \alpha_j v_j$$

$$x(n) = \sum_{j=0}^{m} c_j(0)\lambda_j^n v_j + \sum_{j=0}^{m} \alpha_j v_j$$

$$\frac{x(n)}{P(n)} = \frac{(c_0(0) - \frac{\alpha}{1-\lambda_0})\lambda_0^{-n} + \frac{\alpha_0}{1-\lambda_0}}{c_0(0)\sum_{j=0}^{m} v_0(j) + \lambda_0^{-n}\sum_{j=0}^{m} u_n(j)}.$$

Stable Age Distribution:

$$\frac{x(n)}{P(n)} \to \frac{\alpha_0}{(1-\lambda_0)c_0(0)\sum_{j=0}^{m} v_0(j)}$$

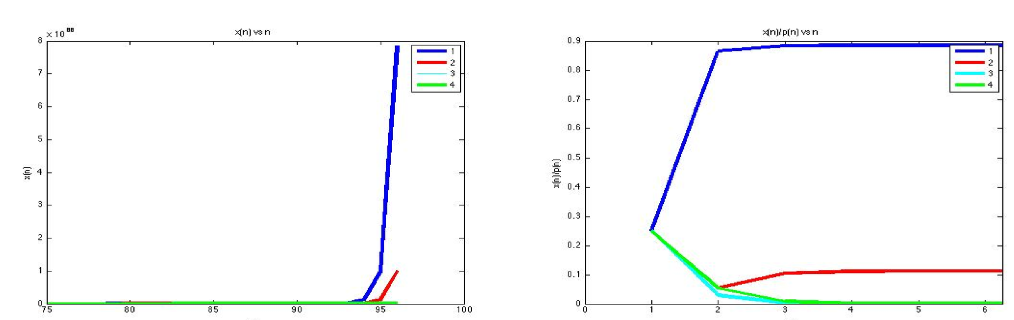
Characteristic Equation:

$$\lambda^4 - a\lambda^3 - p_0 a\lambda^2 - p_1 p_0 a\lambda - p_0 p_1 p_2 a$$

Numerical Experiments

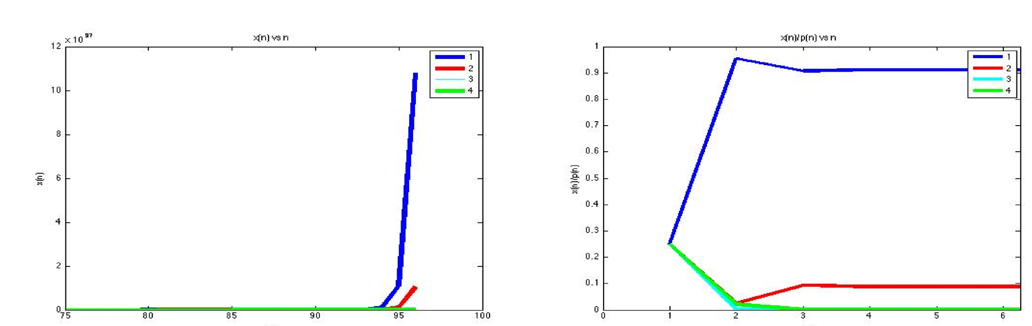
$$x(n+1) = Ax(n)$$

$$A = \begin{bmatrix} a & a & a & a \\ p_0 & q_1 & 0 & 0 \\ 0 & p_1 & q_2 & 0 \\ 0 & 0 & p_2 & q_3 \end{bmatrix} = \begin{bmatrix} 7.0000 & 7.0000 & 7.0000 & 7.0000 \\ 0.9058 & 0.8147 & 0 & 0 \\ 0 & 0.1270 & 0.8147 & 0 \\ 0 & 0 & 0.9134 & 0.8147 \end{bmatrix}$$

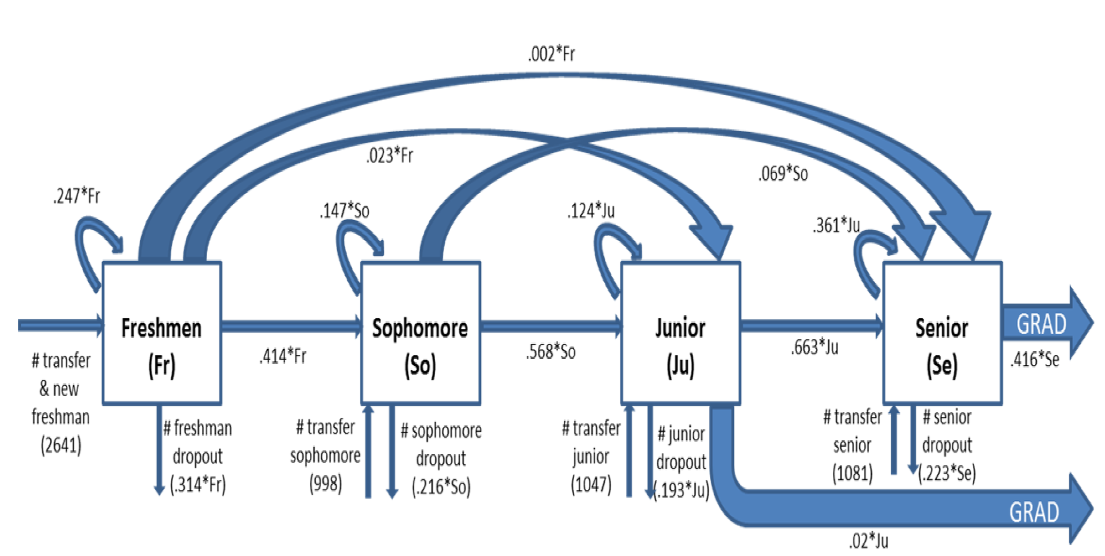


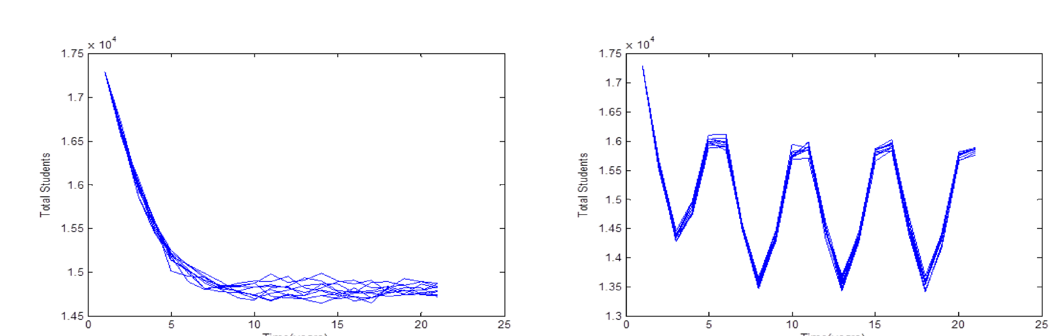
$$x(n+1) = Ax(n) + b$$

$$A = \begin{bmatrix} a & a & a & a \\ p_0 & 0 & 0 & 0 \\ 0 & p_1 & 0 & 0 \\ 0 & 0 & p_2 & 0 \end{bmatrix} = \begin{bmatrix} 9.0000 & 9.0000 & 9.0000 & 9.0000 \\ 0.9619 & 0 & 0 & 0 \\ 0 & 0.0046 & 0 & 0 \\ 0 & 0 & 0.7749 & 0 \end{bmatrix}$$



Markov Model for Enrollment





Year	Markov Chain	Monte Carlo Median	2.5 % quartile	97.5% quartile
Freshman	3789	3789	3729	3847
Sophomore	3446	3448	3368	3518
Junior	3637	3640	3568	3710
Senior	5857	5856	5758	5957
Total Students	16730	16730	16609	16855

Discussion & Future Work

The stable age distribution for the modified Leslie model involves the term α_0 which could change how the population is divided up drastically. These models also need to be applied to data collected from our university and modified to best fit the data. We would also like to apply a term that is time dependent on the population to the vector b mentioned in the modified Leslie model. This would allow for a more realistic situation for modelling enrollment.

References

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3) Fred Brauer and Carlos Castillo-Chavez "Mathematical Models in Population Biology and Epidemiology",New Yory, NY (2010).

4)http://www.tn.gov/education/reports data.shtml.

5)http://hit.state.tn.us/birthrateform.aspx

6)http://www.etsu.edu/opa/fact/factbooks.aspx

Acknowledgements

We were supported by ETSU to attend multiple conferences to present this material. We would like to thank Dr. Cintron-Arias and Dr. Joyner for their assistance in this research. This research was supported by the an Access and Diversity Initiative from the Tennessee Board of Regents, grant #210024.

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