Dynamic Behavior of the Discrete-Time Logistic Model

Jason Beaulieu, Jeremy Brooks, Derek Cassel, Tom Gemmer, William Jamieson, and Wesley Surber

Math 3150: Mathematical Modeling, East Tennessee State University

Mentor: Ariel Cintrón-Arias, Ph.D.

Abstract

The discrete-time logistic model is introduced to describe dynamical behavior of populations. We calculate fixed points and establish stability conditions. We carry out a graphical technique called cobwebbing. We discuss the existence and stability of periodic cycles and their dependence on a model parameter. Numerical simulations are used to obtain an orbital bifurcation diagram for the discrete-time logistic equation. We estimate model parameters from longitudinal observations about the growth of *Paramecium aurelia* in isolation.

Introduction

Many different phenomenon in nature can be described by simple mathematical models. These simple models can display complex behaviors in which we can discover an array of dynamical properties. We can describe the population growth for *P. aurelia* by using the discrete-time logistic equation:

$$x_{n+1} = rx_n\left(1 - {x_n}/{K}\right)$$

Fixed Points and Stability

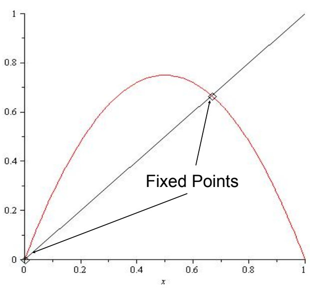

Figure 1: Plot from MAPLE shows fixed points for the logistic equation (in red) with r=3 and K=1. Fixed points are marked with a diamond.

Cobwebbing

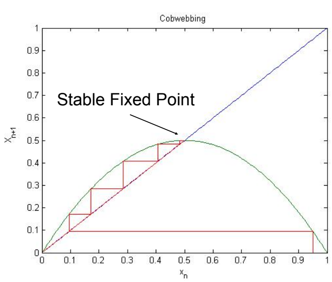

Figure 2: Cobwebbing technique for the logistic equation. x_{n+1} v. x_n for the logistic equation (in green), and $x_{n+1}=x_n$ (in blue), and the cobweb progression (in red).

Period Doubling

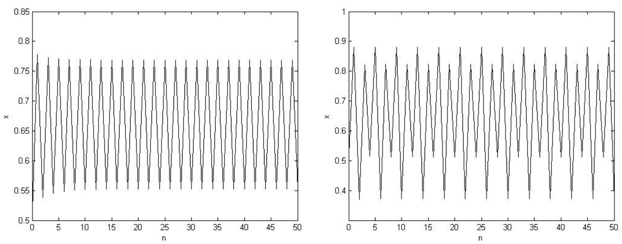

Figure 3: The figure on the left shows a period 2 cycle with *r=3.110*. The figure on the right shows a period 4 cycle with *r=3.521*.

Orbit Diagrams

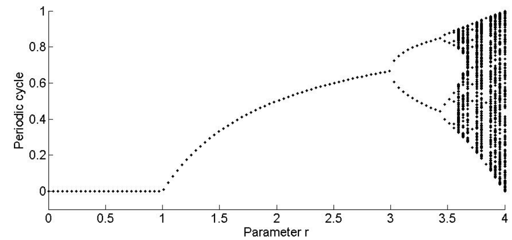

Figure 4: Orbit diagram for the logistic model showing r varying from 0 to 4.

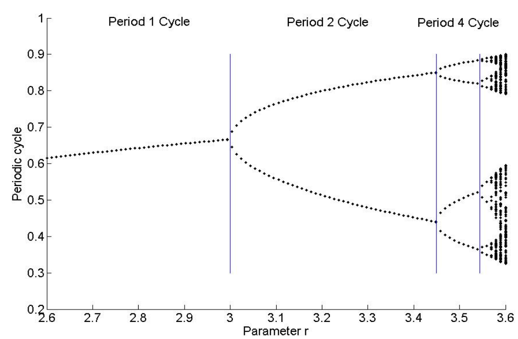

Figure 5: Orbit diagram for the logistic model detailing bifurcation into period 2 and period 4 cycles respectively at $r = 3$ and $r = 1+\sqrt{6}$.

Paramecium aurelia

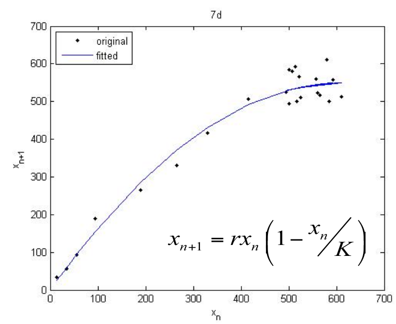

Figure 6: Graph of population data on *P. aurelia*[1] for x_{n+1} vs. x_n fitted to the logistic model. With *r=1.785* and *K=1233.109* as calculated with the statistical package R.

Conclusion

Simple models give rise to complex dynamics. By changing different parameters of the logistic equation we can observe an array of orbits including period-1 cycles, period-2 cycles, and even period-32 cycles. The discrete-time logistic equation can model different population growths as shown by the *Paramecium aurelia* model.

References

1. G. de Vries, T. Hillen, M. Lewis, J. Muller, B. Schonfisch, *A Course in Mathematical Biology: Quantitative Modeling with Mathematical and Computational Methods,* Society for Industrial and Applied Mathematics, Philadelphia, 2006.
2. F. Brauer, C. Castillo-Chavez, *Mathematical Models in Population Biology and Epidemiology,* Springer-Verlag, New York, 2001.
3. R.M. May, Simple mathematical models with very complicated dynamics, *Nature* **261** (1976), 459-467.