Game Theory Analysis of Vaccination Coverage with Epidemic Modeling

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Abstract

We consider vaccination within a population, using game theoretical modeling. Special emphasis is placed on epidemic modeling and the inherent conflict between individual interest and group interest. Our results are subjected to sensitivity analysis using Monte Carlo sampling and are summarized in histograms detailing the difference in coverage and increased mortality. Additionally, the role of perceived risk in changing the population's vaccine uptake level is graphically and analytically explored.

Introduction

In populations with voluntary vaccination policies, the level of vaccine coverage may not reach the level that would be best for the population. We wish to investigate what differences occur by employing a game theory model, where players choose strategies with expected pay-off.

Individual Interest v. Group Interest

With individuals as players, there exist two strategies. Vaccinator and Delayer. The former vaccinates preemptively and the latter declines preemptive vaccination but seeks vaccination during an outbreak. Our model assumes $0 \le p \le 1$ is the proportion of the population playing Vaccinator strategy. We seek a Nash equilibrium so that the pay-off for Vaccinators and the pay-off for Delayers are the same. This is known as the 'Individual Equilibrium'. This is the proportion that is expected to vaccinate in a voluntary system where everyone in the population acts in self interest to maximize their own survival. The pay-off for a Delayer is a function of the proportion playing Vaccinator, so we seek a solution to the equation:

$$E_{vac} = E_{del}(p_{ind})$$

To find the optimum coverage level for the population, we define a 'cost' function that measures mortality associated with playing each strategy. To find the optimum proportion of coverage, we minimize our cost function with respect to p. This new proportion is the level of coverage that minimizes the mortality of the population.

Epidemic Model

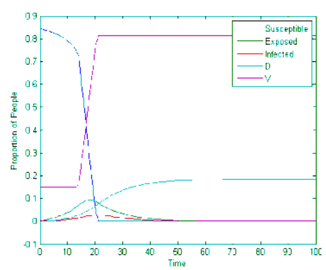
The cost function and Delayer pay-off are both functions of the probability that an unvaccinated individual becomes infected. To find this probability we use a compartmental model. Our model tracks the proportion of individuals who are susceptible to infection (S), exposed to infection but not yet infectious (E), infectious (I), removed from the population by infection (D) and removed due to vaccination (V).

$$\dot{S} = -\beta SI - f(S,t)$$
$$\dot{E} = \beta SI - \sigma E$$
$$\dot{I} = \sigma E - \gamma I$$
$$\dot{D} = \gamma I$$
$$\dot{V} = f(S,t)$$

$$f(S,t) = \begin{cases} 0 & \text{if } 0 \le t \le t_{res} \\ \nu & \text{if } t \ge t_{res} and\ S > 0 \\ 0 & \text{if } t \ge t_{res} and\ S = 0 \end{cases}$$

Our compartment D houses the proportion of the population that was infected. Using the proportion in D as our probability of infection we can solve our Individual Equilibrium and Group Optimum.

Numerical Solution to Epidemic Model

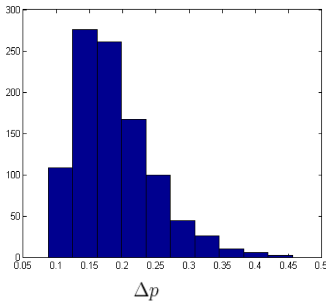


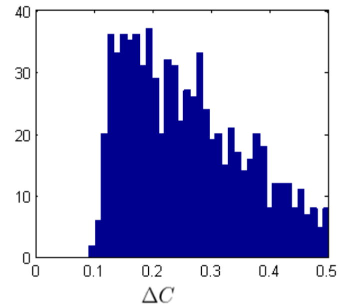
Difference in Coverage and Mortality

The two quantities we will be most interested in will be the difference in coverage and the relative difference in mortality.

$$\frac{\Delta C}{C} = \frac{\left(C(p_{ind}) - C(p_{gr})\right)}{C(p_{gr})}$$

$$\Delta p = p_{gr} - p_{ind}$$





We use Monte Carlo sampling to vary the parameters on intervals and show our results in histograms. The distribution for Δp is centered between 0.15 and 0.20. This indicates that is it very unlikely that individuals vaccinating voluntarily in self-interest will attain optimum coverage for the group. Our relative difference in cost, ΔC, has a distribution centered between 0.13 and 0.25. This corresponds to an increase in mortality by 13-25% because of sub-optimal vaccination coverage.

Perceived Risk

Perceived risk plays a critical role in our model, such that even a slight risk associated with vaccination may outweigh the risk of infection from an individual's point of view. We consider a relative risk $r = r_v/r_i$ where r_v is the perceived risk from vaccination and r_i is the perceived risk from infection.

New Player Strategies

Players now choose the strategy of vaccinating with probability P or Q. Each strategy has an expected pay-off, E_P. The expected change in pay-off for switching strategies can be expressed as

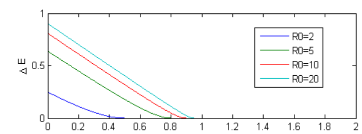
$$\Delta E = E_P - E_Q$$

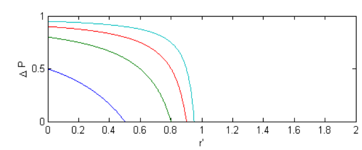
For any perceived risk, there exists a unique strategy $P = P^*$ such that ΔE is strictly positive for all $Q \neq P^*$.

Change in Perceived Risk

For any increase in perceived risk, $r' > r$, there is a corresponding change in pay-off and vaccine uptake levels. The degree to which r' is likely to have an impact on vaccination behavior is sensitively dependent on the value of R_0, the basic reproductive number. We define the difference in vaccination uptake as $\Delta P = P_{r'} - P_r$ where $P_{r'}$ and P_r are the vaccine uptake levels of the population corresponding to each relative risk.

While $r' > 1$, vaccination is perceived to be more risky than infection so a "never vaccinate" equilibrium persists. When $r' < 1$, the vaccine uptake and incentive for switching strategies both increase at a rate dependent on the reproductive ratio of the infection.





Conclusion

Using a "vaccination game" we quantitatively show the effect of individual's choice on the mortality of infections and the overall vaccine uptake levels of a population. These models can provide new insights for vaccination policy.

References

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