Prisoner's Dilemma Implementation on Watts Strogatz Networks and Real Networks

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Abstract

Prisoner's Dilemma is a game theoretical model that considers two opposing strategies: Cooperation and Defection. Cooperators play an altruistic strategy, while defectors play a selfish strategy. In a two-player game and in a homogeneous population, we see defectors always winning or invading the population because they never have to pay any cost, which causes Defectors to have a higher payoff and a higher fitness, Figure 1. However, biologically, we see the Cooperation strategy persisting in small clustered communities because individuals realize that helping others increases the probability that they will receive help from those individuals later, which in turn should yield a higher payoff. Here we investigate the effects space has on the Prisoner's dilemma game by simulating it on the Watts-Strogatz modeled networks and investing networks obtained from real data, namely the Facebook friendship data obtained from the California Institute of Technology.

$$\frac{dx_D}{dt} = x_D(f_D - \Phi)$$

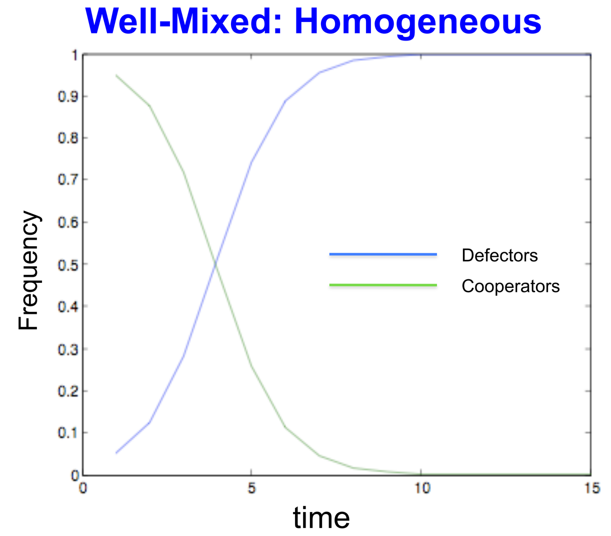
$$\frac{dx_C}{dt} = x_C(f_C - \Phi)$$

$$\Phi = x_C f_C + x_D f_D$$

$$f_C = (b-c)x_C - c x_D$$

$$f_D = b x_C$$

Well-Mixed: Homogeneous



Introduction

Prisoner's Dilemma is a game theoretical model that describes interactions between two strategies: Cooperation and Defection. Cooperator's are altruistic individuals who help others by giving a benefit (b) with no guarantee that they will receive any benefit in return, which is the risk or cost (c) that they pay in every interaction. Defectors play the selfish strategy and only take from giving Cooperators, while not reciprocating any benefit and therefore not having to pay any cost. In a homogeneous model, where every individual has equal probability of interacting with every other individual, we see that Defectors have a great advantage over the Cooperators because they never have to pay any cost. The payoff matrix for the two player game is as follows: **A** =

$$\begin{array}{c|cc} & C & D \\ \hline C & b-c & -c \\ D & b & 0 \end{array}$$

In a spatial model, individuals may interact with more than one other individual in a single time step and occur between neighbors. In a network, we let nodes represent individual players and edges between nodes represent neighbors that have the potential to interact in each time step. Therefore, when taking into account space, we let (i) represent the number of Cooperating neighbors, while (k) is the total number of neighbors. Then, the payoff of a Cooperating node is bi-ck and the payoff of a defecting node is bi. This can be written as payoff of node i at time t: $P(i,t) = \sum_{j \varepsilon \Omega_i} x_i^T A x_j$

where $x_i = \begin{bmatrix}1\\0\end{bmatrix}$ for Cooperators and $x_i = \begin{bmatrix}0\\1\end{bmatrix}$ for Defectors

A is the payoff matrix between two players, shown above and Ω is the set of neighbors of node i. Fitness is then calculated by 1-w+wP(i,t), where w is intensity of selection.

Watts-Strogatz networks are networks that start out with the same degree for every node, which initially creates a ring. (p) represents the rewiring probability, meaning that in each time step , the actual network is updated, an edge is chosen at random to be rewired to a new node with probability (p). With p=0, we have a ring and as p approaches 1, the graph tends toward and Erdos-Reyni graph. This creates small clustered communities, while maintaining small average path length. Using algorithms written in Matlab, we can create a small world network, vary the rewiring probability, simulating prisoner's dilemma on the network, and run the implementation of prisoner's dilemma on any adjacency matrix obtained from real data.

Model

This model is an evolutionary model, which means it changes through time. When updating this game, we use two methods of updating: death-birth updating and imitation. Death-birth updating kills a node or player in every time step and the nodes connected to it compete for the empty site proportional to their fitness. Fitness is calculated by 1-w+wp where w is intensity of selection (here w=1 always) and p is the payoff of the node. The payoff of each node is calculated subjective to the strategy of the node. Cooperators have a payoff bi-ck, where (i) is the number of cooperating neighbors and (k) is the degree of the node. Similarly, the payoff of a defector is bi. Figure 1 illustrates a hypothetical example of this updating. Yellow nodes represent cooperators, while black nodes represent defectors. The ? Node is the node chosen at random to die, and nodes C and D are competing for the empty site in the next time step. For the following simulations, a Watts-Strogatz network with rewiring probability equal to 0 (a ring) was used to play Prisoner's Dilemma based on this rule. For the imitation updating, a node is chosen at random to become active, then one of the neighboring nodes of the active node is chosen at random and the active node simply takes the strategy of that node.

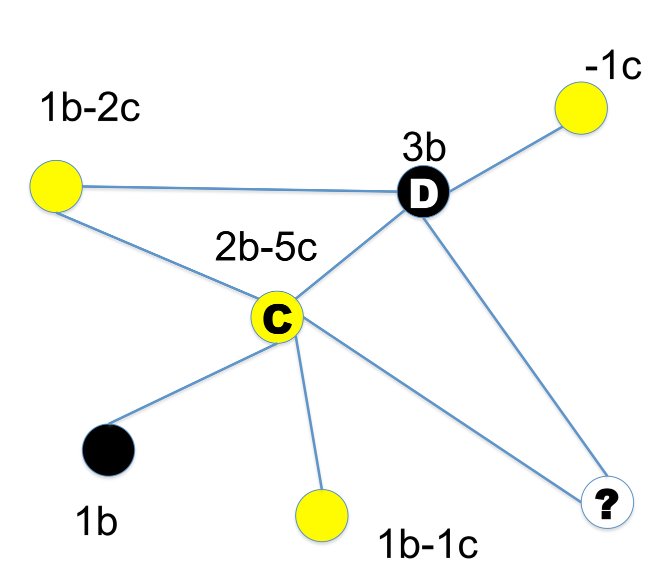


Figure 2: Illustrates an example of Death-Birth updating. Node (?) is the empty site, while C and D are competing to take over proportional to their fitness.

Watts-Strogatz p = 0: Death-Birth Updating

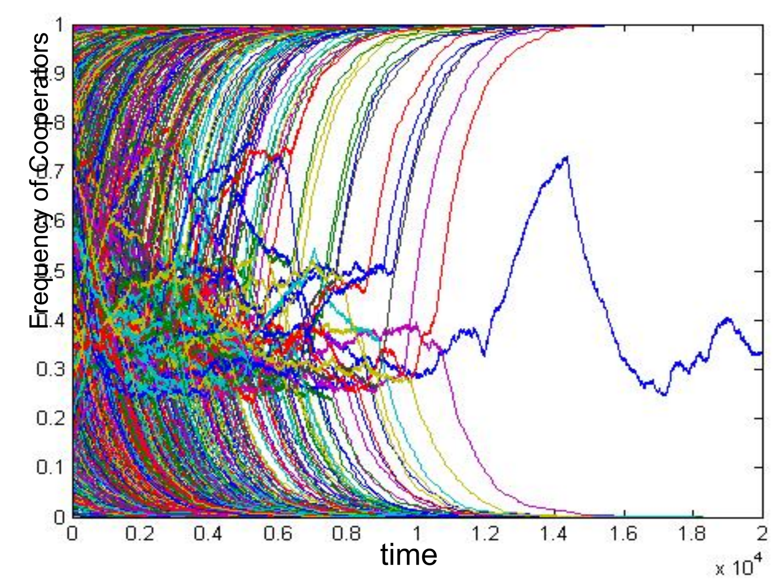


Figure 3: Plots the number of Cooperators in the network against time. The parameters are b = 0.6 and c = 0.3. Each color represents a different realization (1,000).There are 20,000 time steps. This is for the Watts-Strogatz network with p = 0, where average degree is 4.

Watts-Strogatz p = 0: Death-Birth Updating

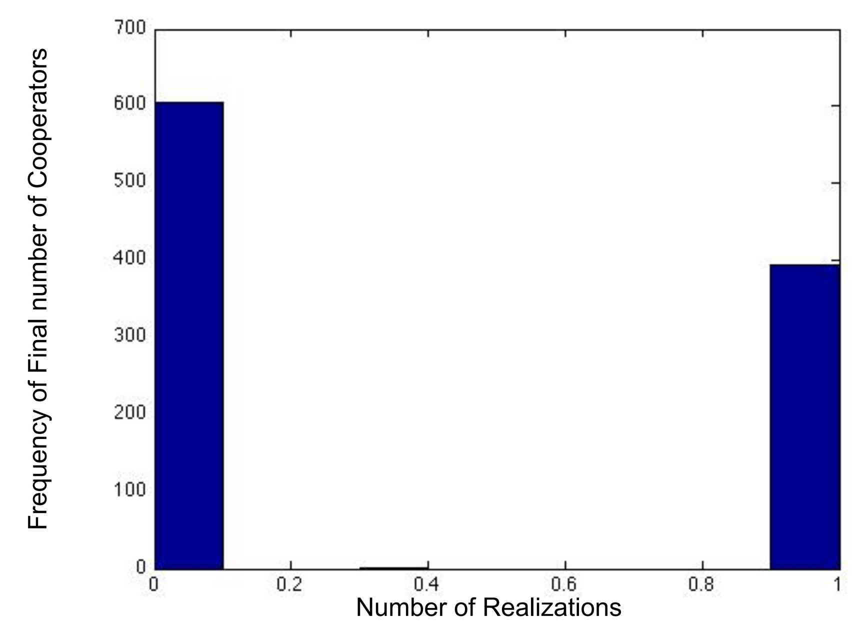


Figure 4: Histogram of the final number of Cooperators in the network against frequency. This is for the Watts-Strogatz network with p =0, where average degree is 4 and corresponds to Figure 3.

Watts-Strogatz p = 0: Imitation Updating

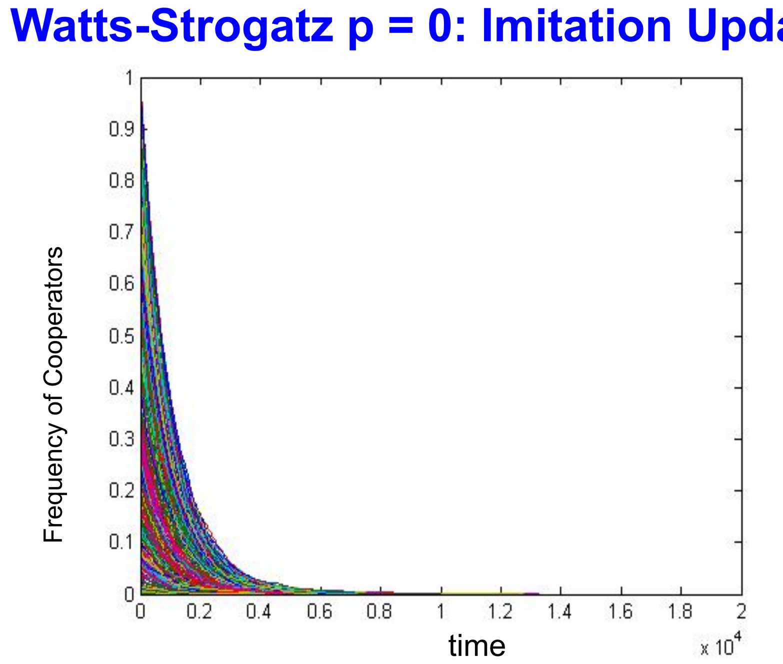


Figure 5: Plots the number of Cooperators in the network against time. The parameters are b = 0.6 and c = 0.3. Each color represents a different realization (1,000), each having 20,000 time steps. This is for the Watts-Strogatz network with p =0, where average degree is 4.

Watts-Strogatz p = 1: Death-Birth Updating

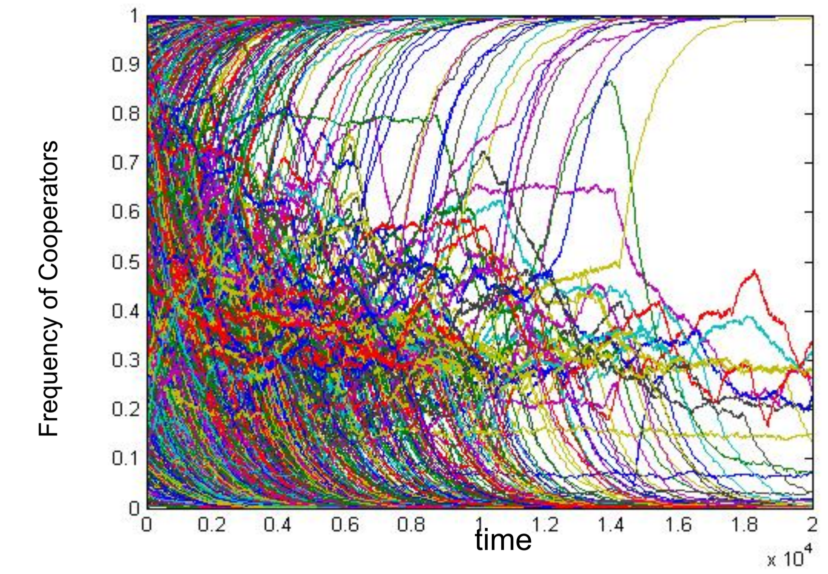


Figure 7: Plots the number of Cooperators in the network against time.The parameters are b = 0.6 and c = 0.3. Each color represents a different realization (1,000), each having 20,000 time steps.. This is for the Watts-Strogatz network with p =1, where average degree is 4.

Watts-Strogatz p = 1: Death-Birth Updating

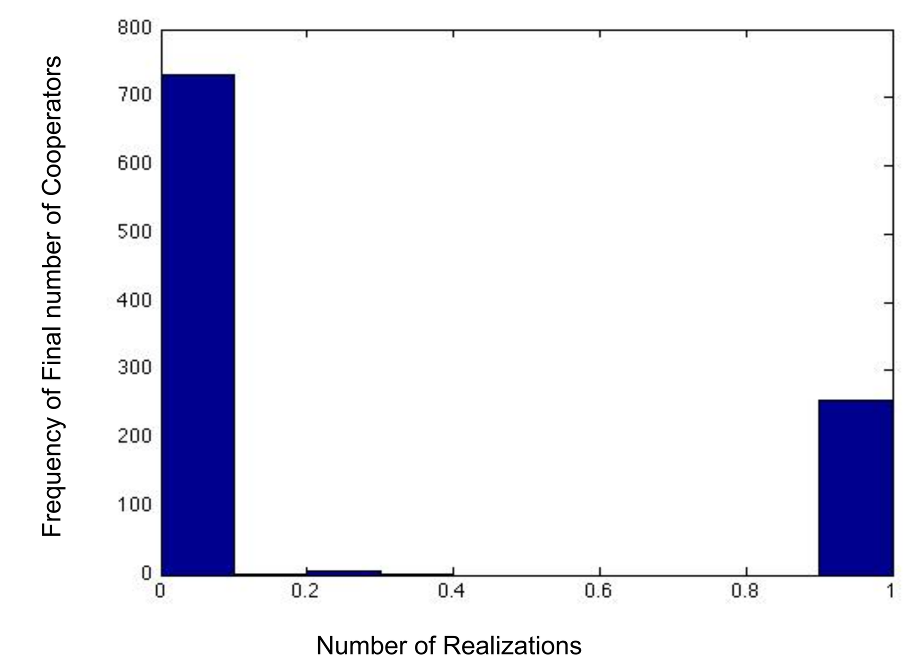


Figure 8: Histogram of the final number of Cooperators in the network against frequency. This is for the Watts-Strogatz network with p =1, where average degree is 4, and this corresponds to Figure 7.

Watts-Strogatz P = 1: Imitation Updating

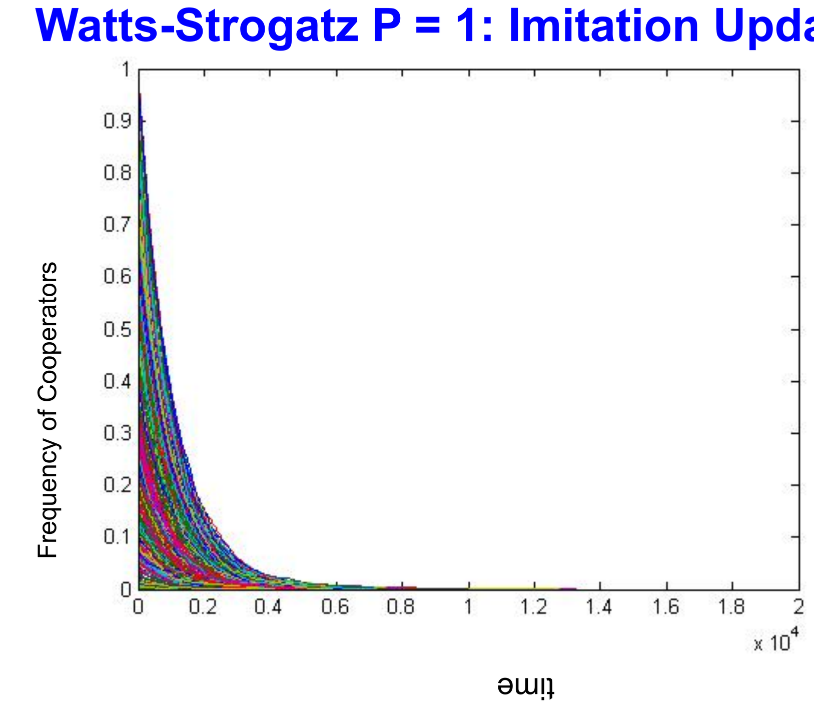


Figure 9: Plots the number of Cooperators in the network against time. The parameters are b = 0.6 and c = 0.3. Each color represents a different realization (1,000),each having 20,000 time steps. This is for the Watts-Strogatz network with p =1, where average degree is 4.

Cal-Tech: Death-Birth Updating

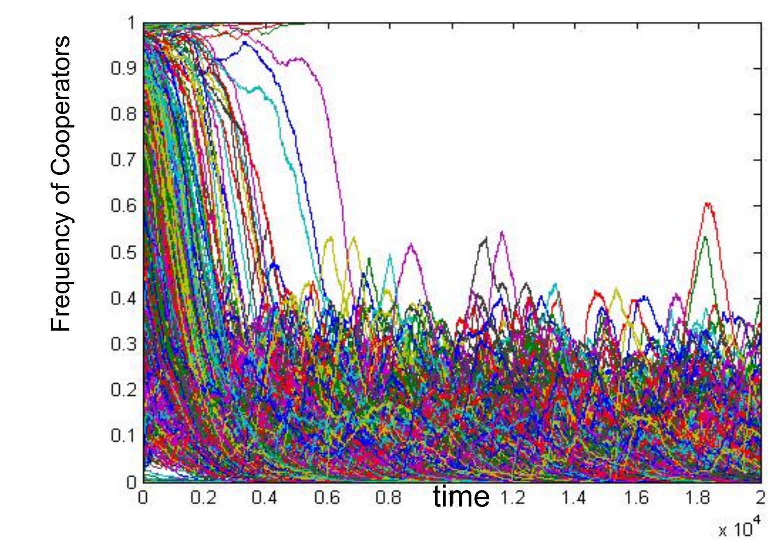


Figure 11: Plots the number of Cooperators in the network against time. The parameters are b = 0.6 and c = 0.3. Each color represents a different realization (1,000), with 20,000 time steps each. This is from a network created from the facebook social network at Caltec.

Cal-Tech: Death-Birth Updating

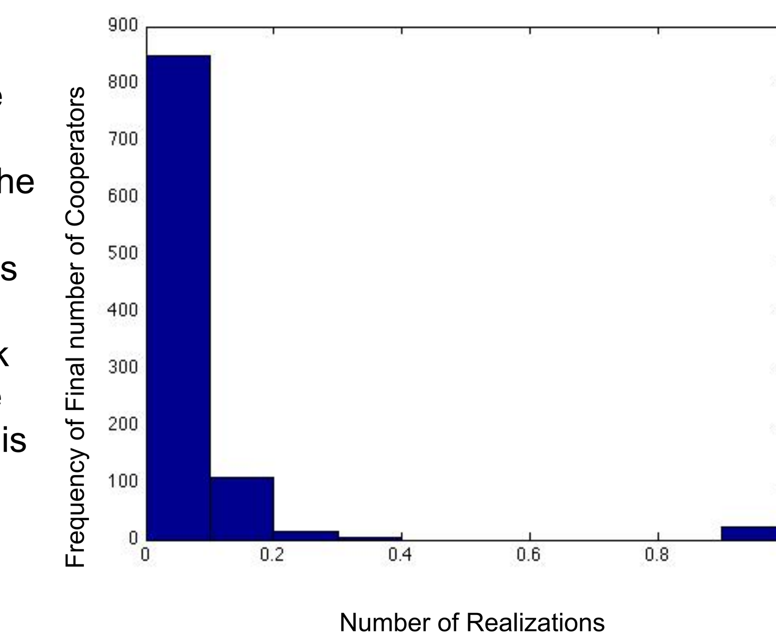


Figure 12: Histogram of the final number of Cooperators in the network against frequency. This is for the Caltech data and corresponds to Figure 11.

Cal-Tech: Imitation Updating

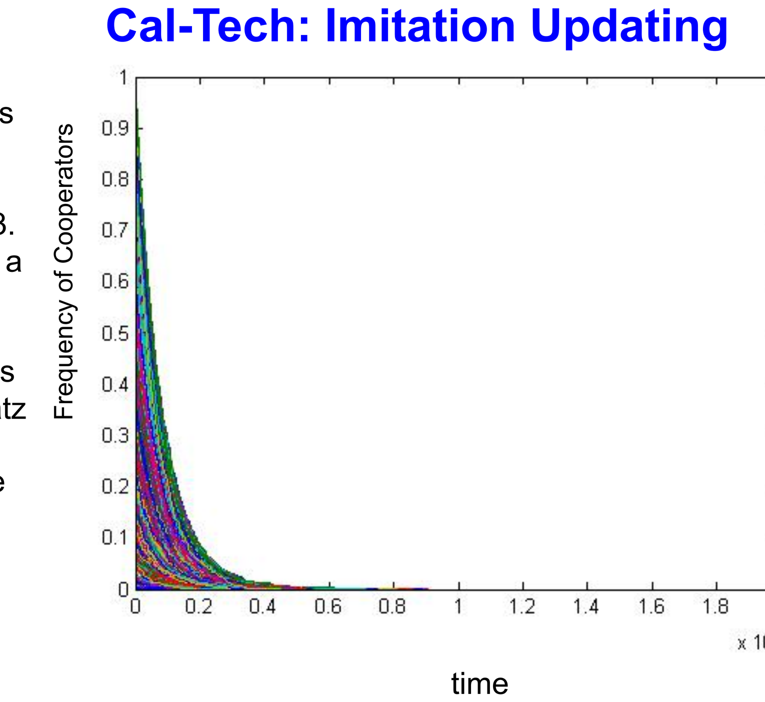


Figure 13: Plots the number of Cooperators in the network against time. The parameters are b = 0.6 and c = 0.3. Each color represents a different realization (1,000), where each realization has 20,000 time steps. This is for the Caltech data using imitation updating.

Conclusion

In a well-mixed population, Cooperators go extinct in relatively few times steps. When taking into account the affect space has on a population, we see that b/c > <k> is a necessary condition for Cooperators to prevail. Playing Prisoner's Dilemma on a graph generated from a network created with real data gives us insight as to why altruistic behavior persists throughout a population, even though the strategy is seemingly not beneficial to the individual. We see in both the Watts-Strogatz networks and Caltch data that Cooperators either completely invade or go extinct. However, there are a few cases where they co-exist in the population. In the imitation updating, since fitness has no say in the mimicking and everything is left to chance, it makes sense to see Cooperators dying out. Thus, the network structure and updating methods both have drastic effects on what seems to be fittest strategy in the network.

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