Evolutionary Dynamics Approach to Vaccination On Social Network Structure

Jordan Angel†, Samuel Peters† and Manuel Gonzalez††

East Tennessee State University Department of Mathematics and Statistics†, East Tennessee State University College of Public Health††

ABSTRACT

Previous research has considered three strategy games with vaccinators, delayer and non-vaccinator strategies.[1] These models assumed well-mixed populations with no underlying structure. We attempt to extend this model to a social network structure using agent-based simulations with small-world network structure. We employ an evolutionary game theory model on small-world networks. The game theory model is coupled with an age-structured epidemiological model that determines the payoff associated with the strategies.

INTRODUCTION

Game theory provides an abstract formulation of a decision situation that may have conflicting interests. These decisions correspond to payoffs for the players choosing them. Evolutionary game theory says that strategies have a fitness that is a measure of how successful a strategy is given the state of the population. A well-studied general three strategy game is Rock-Paper-Scissors. This game has the following payoff matrix,

$$\begin{array}{c} \\ R \\ P \\ S \end{array} \begin{array}{c} \begin{array}{ccc} R & P & S \end{array} \\ \begin{pmatrix} 0 & -l & w \\ w & 0 & -l \\ -l & w & 0 \end{pmatrix} \end{array}. \tag{1}$$

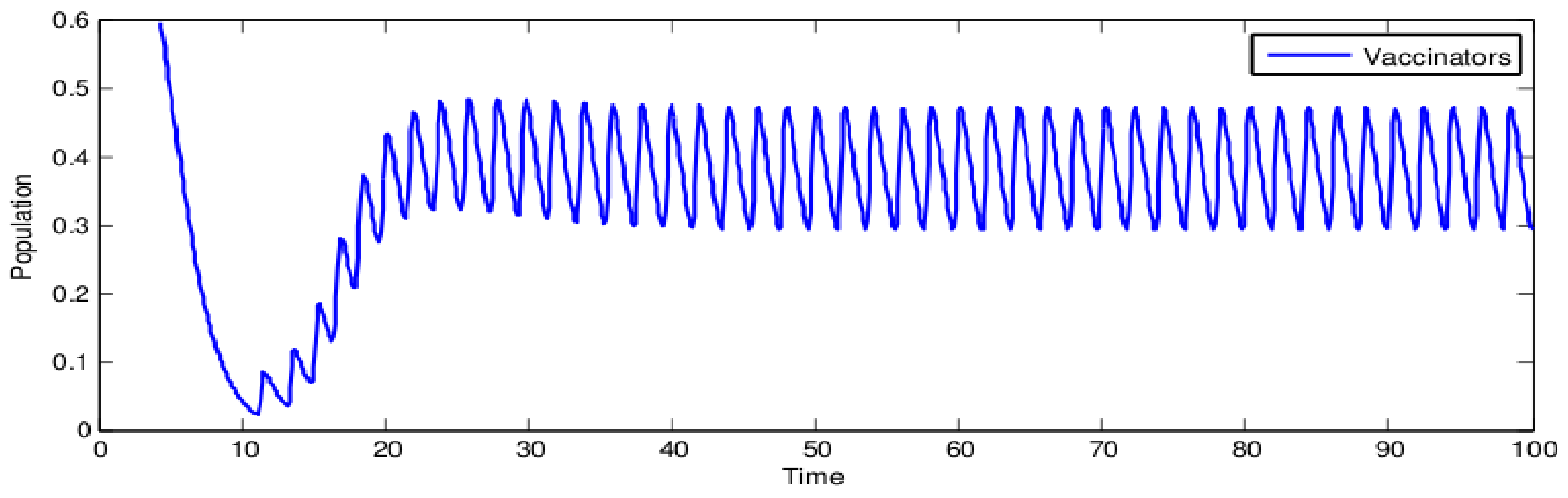
If we have $w > l$ and we represent the proportion playing each strategy as a vector $x = (x_1, x_2, x_3)$, then $x^* = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is said to be an evolultionary stable strategy, meaning small deviations from x^* will converge back to x^* . If $w = l$, x^* is neutrally stable so that deviations from x^* can persist but are not likely to grow. Finally, for $w < l$, x^* is neither. The fitness of the strategies is found by taking the product of the payoff matrix and strategy vector,

$$\begin{bmatrix} 0 & -l & w \\ w & 0 & -l \\ -l & w & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 w - x_2 l \\ x_1 w - x_3 l \\ x_2 w - x_1 l \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \tag{2}$$

Utilizing this concept of fitness, we can write a replicator equation that approximates the growth of each strategy. We write this equation as,

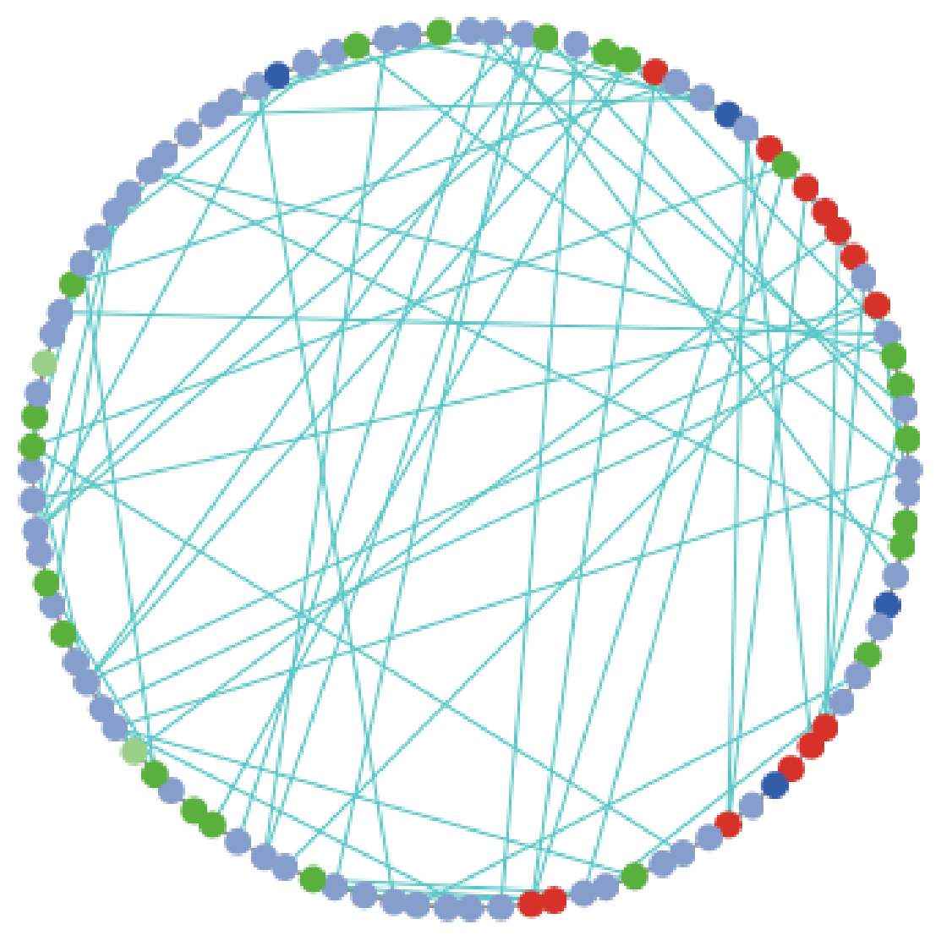
$$\dot{x_i} = x_i(f_i - \phi) \tag{3}$$

where ϕ is the average fitness given by $\phi = \sum_{i=1}^{n} x_i f_i$ Using these replicators equations we can examine the numerical approximations to the game outcome. Here is an example outcome for the strategy of vaccinators. After a short time the system settles into a limit cycle. This is a closed orbit in the strategy phase space.

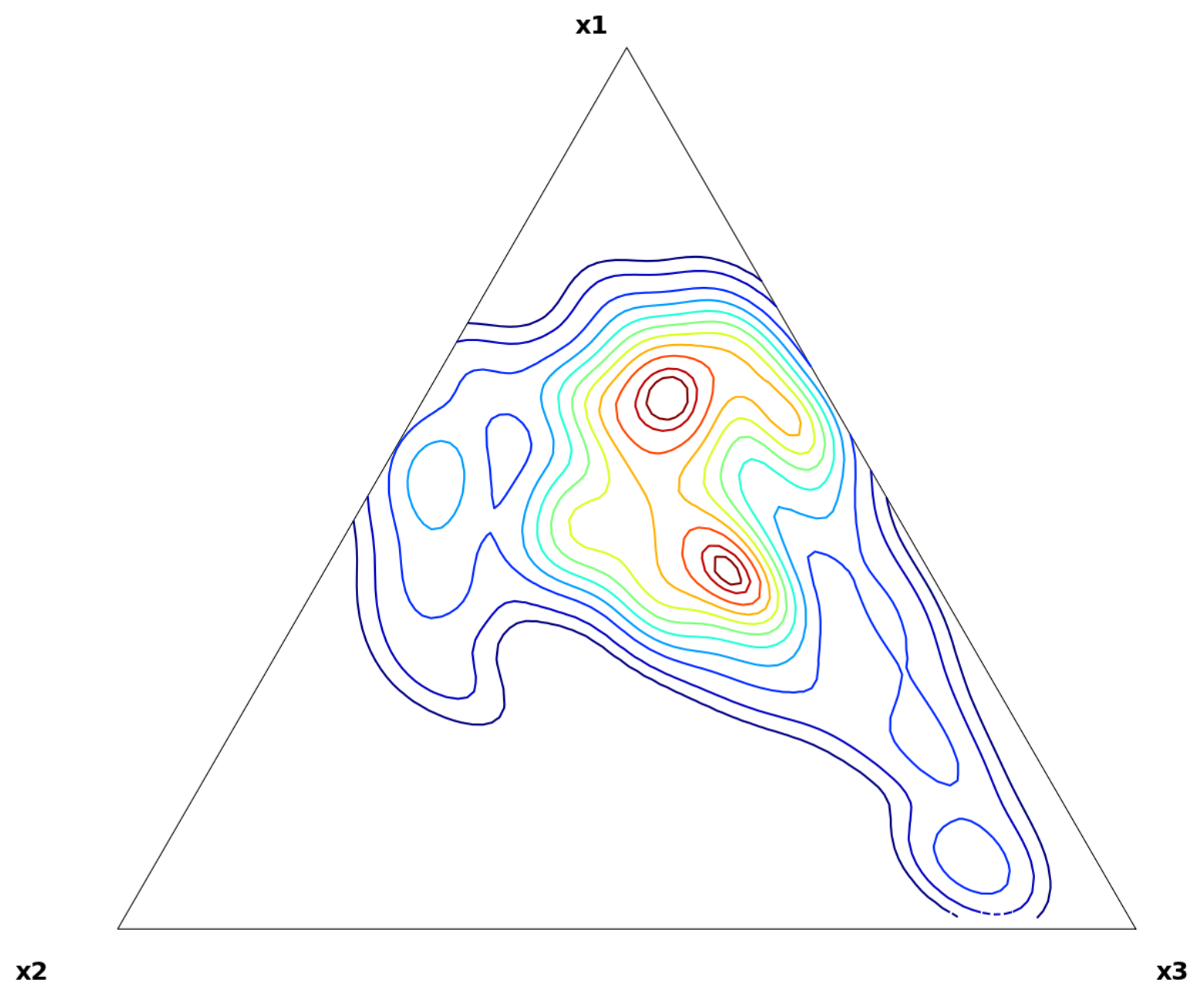


MODEL

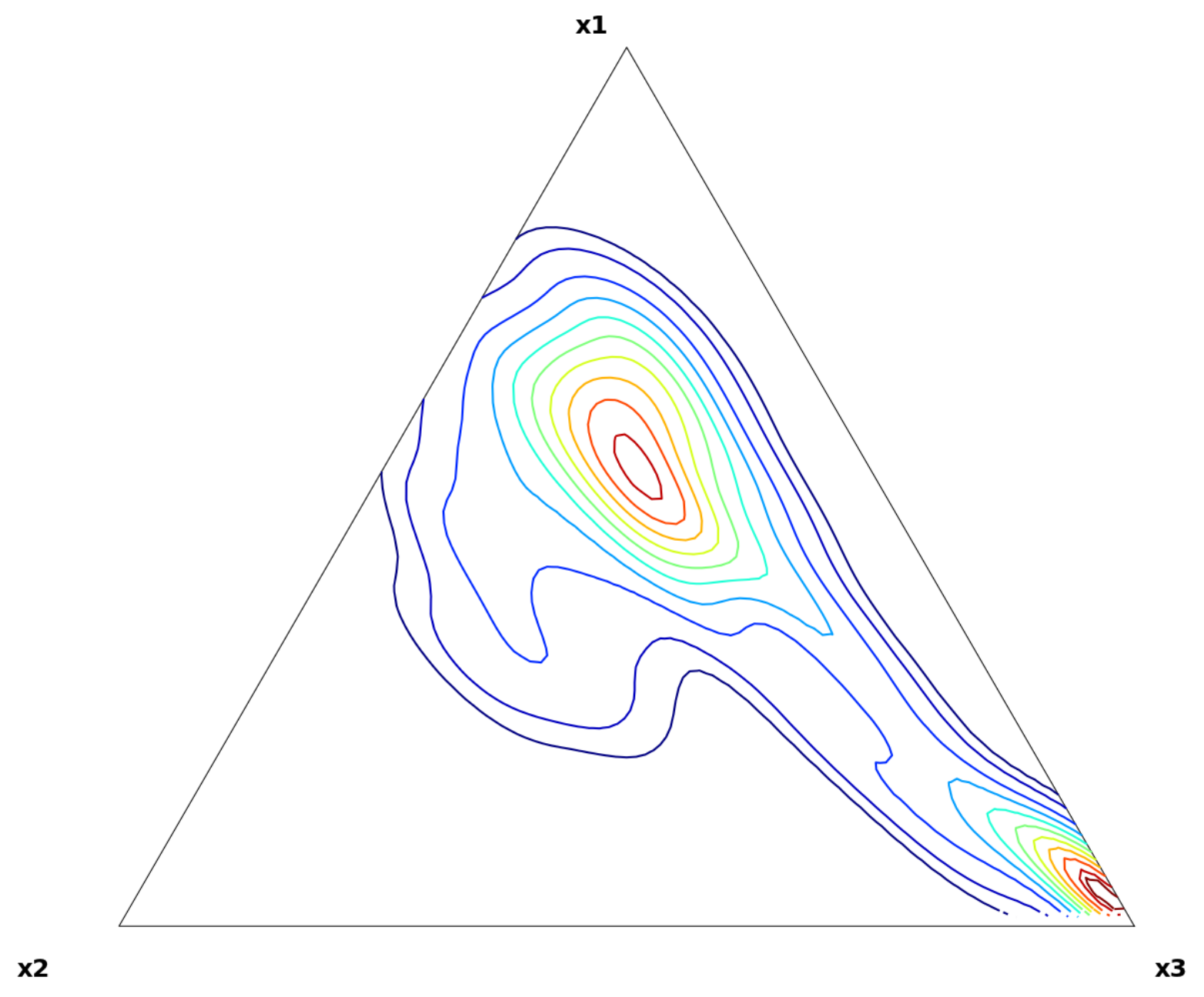
Agents are represented as nodes on a small-world network. Small-world networks are characterized by having short average path length and high clustering coefficient. Structurally this means the average path from any node to another is relatively short and there tends to be pockets of nodes that are well connected to each other. These traits are found in social networks. The game theory model is coupled with an age-structured SIR model. Payoffs for delayers and non-vaccinators are functions of the number of infected persons which an agent shares a connection. To update the game, we use the Moran Process. An agent is randomly selected to be replaced and the new agents strategy is determined in proportion to each strategies fitness. We use Best Response rule so that new agents sample their neighbors and select the strategy with the greatest fitness.



RESULTS



Endemic Infection

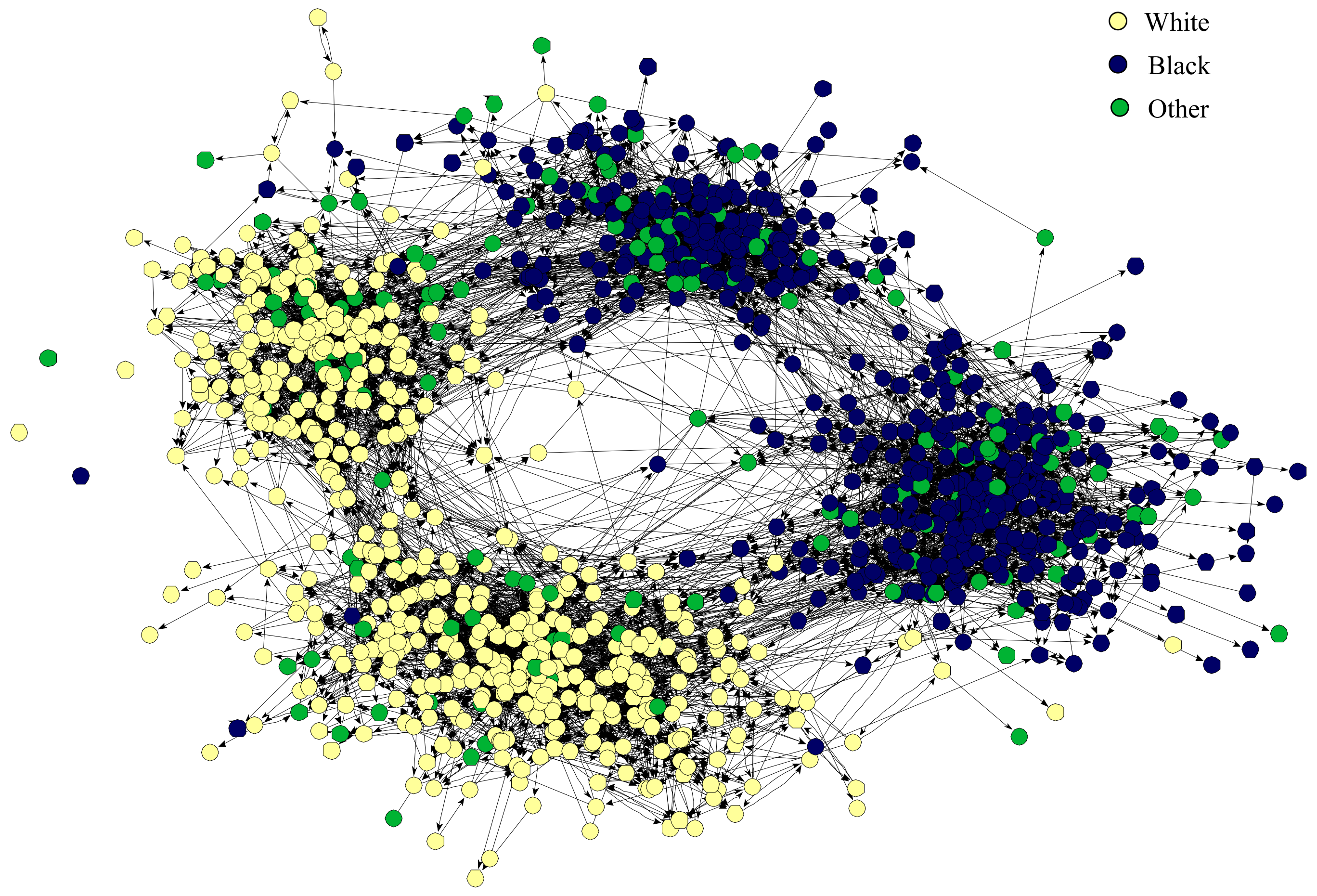


Extinct Infection

DISCUSSION

The outcome of the game is critically dependent on the epidemilogical model. As infection oscillates in the population the payoffs oscillate as well. If the disease is endemic, a mixed population strategy exists and undergoes perturbations that occur from changes in the payoff matrix. However if the disease goes extinct, the only solution becomes $x^* = (0, 0, 1)$ or a pure non-vaccinator strategy. Agents adjust their payoff based on the number of infected agents which with they share a connection. This limited amount of information can cause agents to overestimate and underestimate the actual disease prevelance. This causes agents to overcompensate their strategy leading to greater flucuations in the populations strategy as a whole.

It is the ultimate goal of this project to run similar simulations on real social network data such as National Longitudinal Study of Adolescent Health shown below.



REFERENCES

[1] S. Bhattacharrya, C. T. Bauch *A game dynamic model for delayer strategies in vaccinating behaviour for pediatric infectious diseases*, Journal of Theoretical Biology, **267** (2010), 276-282

[2] M. A. Nowak *Evolutionary Dynamics: Exploring the Equations of Life*

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