Epidemiological Processes on Random Graphs

Wesley Surber

Honors-in-Discipline Program, Department of Mathematics and Statistics

East Tennessee State University

Mentor: Ariel Cintrón-Arias, Ph.D.

Abstract

A random graph is a graph that is generated with a given amount of vertices and then edges are added to make connections between the vertices at random. I will be using the Poisson distribution to create the edges of the vertices. Using this random process creates certain graph properties: degree distributions, clustering, and network correlations. Properties of graphs also include their resilience or how resistant they are in keeping connections and eccentricity even after deleting edges and community structure which relates to clustering. Random graphs provide models of community structure to study an epidemiological process. I will be using a SLAIR model to simulate an infectious disease on communities with random graph structure. The SLAIR model contains five different epidemiological states that a vertex can have: susceptible, latent, asymptomatic, infected, and recovered. I will compare the results of the random graphs to the deterministic values of the pair approximations of the graph.

Graph

A graph is a representation of information that can exist as set vertices and edges connecting those vertices.

Types of Random Graphs

- •Erdos-Renyi random graph is produced by creating n vertices and have a probability of p that each vertex is connected.
- •Caveman is a type of random graph that is created to simulate clusters of a population.
- •Lattice is a random graph that is used to simulate an even mixed population.
- •A preferred attachment graph can simulate a social network because the edges are more likely to connect when it has more edges.

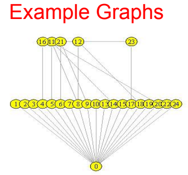
Properties of Random Graphs

Clustering: The density of a certain number of vertices usually determined by the amount of triangles that exist.

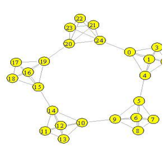
Eccentricity: The maximum number of edges it takes to get to any other vertex.

Diameter: Maximum eccentricity.

Example Graphs

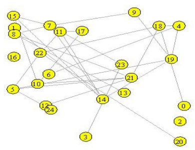


Preferred Attachment

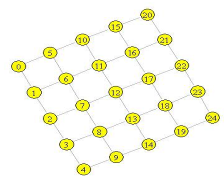


Caveman Graph

Example Graphs

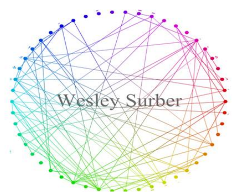


Erdos-Renyi



Lattice

This is a sample graph taken from the social network Facebook using the application Friendship Wheel.

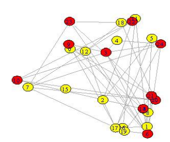


In the center is my name which is connected to the 50 friends on the outside. When two vertices have an edge connecting them in this example, that means that the vertices are friends with each other.

Basic SIS Model

The SIS model is one of the most basic models that can be applied to a graph. Each vertex is only able to exist as one of two states, either susceptible or infected. In each state, there is certain rates at which the vertex will change states.

In this model, the susceptible will change into an infected by the infection rate if there is a connection between a susceptible and an infected, while the infected will enter the susceptible state at a constant recovery rate.

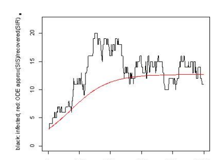


Infection Rate-.05

Recovery Rate-.1

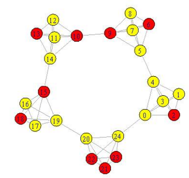
Verticies-25

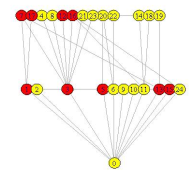
Probability of Connection-.15



This is a single stochastic realization of an SIS model on the associated Erdos-Renyi graph. The red line is the deterministic model of the graph while the black is the stochastic realization.

More Examples of SIS Model





These are the preferred attachment graph and caveman graph that a single realization of the SIS model has been applied to. The red vertices are the Infected while the yellow are still susceptible.

Advanced Models

Another Model that exists is the SLAIR model which is a model with the components of susceptible, latent, asymptomatic, infected , and recovered.

In this model, the latent is infected but not contagious while the infected are infected and contagious and the asymptomatic is infected, contagious, and shows no symptoms of the disease.

Pair Approximations

$[SS]$	$[SL]$	$[SA]$	$[SI]$	$[SR]$
$[LS]$	$[LL]$	$[LA]$	$[LI]$	$[LR]$
$[AS]$	$[AL]$	$[AA]$	$[AI]$	$[AR]$
$[IS]$	$[IL]$	$[IA]$	$[II]$	$[IR]$
$[RS]$	$[RL]$	$[RA]$	$[RI]$	$[RR]$

These are the states which edges can exist and the states which individual or edges can change into.

Event	Transition
Birth	$[S],[L],[A],[I],[R] \Rightarrow [S]$
Death of Susceptible	$[S] \Rightarrow [S]$
Death of Latent	$[L] \Rightarrow [S]$
Death of Asymptomatic	$[A] \Rightarrow [S]$
Death of Infected	$[I] \Rightarrow [S]$
Death of Recovered	$[R] \Rightarrow [S]$
Infection with Asymptomatic	$[SA] \Rightarrow [LA]$
Infection with Infected	$[SI] \Rightarrow [LI]$
Latent to Asymptomatic[SI]	$[L] \Rightarrow [A]$
Latent to Infected[SI]	$[L] \Rightarrow [I]$
Asymptomatic to Recovered	$[A] \Rightarrow [R]$
Infected to Recovered	$[I] \Rightarrow [R]$

$$\frac{d[S]}{dt} = \mu([L]+[A]+[I]+[R]) - \beta([SI]+[SA])$$

$$\frac{d[L]}{dt} = -\mu[L] + \beta([SI]+[SA]) - p\kappa[L] - (1-p)\kappa[L]$$

$$\frac{d[A]}{dt} = -\mu[A] + p\kappa[L] - \eta[A]$$

$$\frac{d[I]}{dt} = -\mu[I] + (1-p)\kappa[L] - \alpha[I]$$

$$\frac{d[R]}{dt} = -\mu[R] + \eta[A] + \alpha[I]$$

$$\frac{d[SS]}{dt} = 2\mu([SA]+[SL]+[SI]) + [SR]) - 2\beta(\frac{[SS][SI]}{[S]} + \frac{[SS][SA]}{[S]})$$

$$\frac{d[SL]}{dt} = \mu([LL]+[AL]+[IL]+[RL]-[SL]) + \beta(\frac{[SS][SI]}{[S]} + \frac{[SS][SA]}{[S]}) - p\kappa[SL$$

$$\cdots - (1-p)\kappa[SL] - \beta(\frac{[LS][SI]}{[S]} + \frac{[LS][SA]}{[S]})$$

$$\frac{d[SA]}{dt} = \mu([LA]+[AA]+[IA]+[RA]-[SA]) + (1-p)\kappa[SL] \ldots$$

$$\cdots - \eta[SA] - \beta(\frac{[IS][SA]}{[S]} + \frac{[AS][SA]}{[S]} + [SA])$$

$$\frac{d[SI]}{dt} = \mu([LI]+[AI]+[II]+[RI]-[SI]) + p\kappa[LS] - \alpha[SI] - \beta(\frac{[IS][SI]}{[S]} \ldots$$

$$\cdots + \frac{[AS][SI]}{[S]} + [SI])$$

$$\frac{d[SR]}{dt} = \mu([LR]+[AR]+[IR]+[RR]-[SR]) + \eta[SA] + \alpha[SI] - \beta(\frac{[AS][SR]}{[S]} \ldots$$

$$\cdots + \frac{[IS][SR]}{[S]})$$

$$\frac{d[LL]}{dt} = -2\mu[LL] + 2\beta(\frac{[LS][SA]}{[S]} + \frac{[LS][SI]}{[S]}) - 2p\kappa[LL] - 2(1-p)\kappa[LL]$$

$$\frac{d[LA]}{dt} = -2\mu[LA] + \beta(\frac{[AS][SI]}{[S]} + \frac{[AS][SA]}{[S]} + [SA]) - p\kappa[LA] \ldots$$

Continued Pair Approximation

$$\frac{d[LI]}{dt} = -2\mu[LI] + \beta([ISI]+[ASI]+[SI]) + 2(1-q)\kappa[LL] - p\kappa[LI] \ldots$$

$$\cdots - (1-p)\kappa[LI] - \alpha[LI]$$

$$\frac{d[LR]}{dt} = -2\mu[LR] + \beta([ISR]+[ASR]) + \alpha[LI] \ldots$$

$$\cdots + \eta[LA] - p\kappa[LR] - (1-p)\kappa[LR]$$

$$\frac{d[AA]}{dt} = -2\mu[AA] + 2p\kappa[LA] - 2\eta[AA]$$

$$\frac{d[AI]}{dt} = -2\mu[AI] + p\kappa[LI] + (1-p)\kappa[LA] - \eta[AI] - \alpha[AI]$$

$$\frac{d[AR]}{dt} = -2\mu[AR] - \eta[AR] + p\kappa[LR] + 2\eta[AA] + \alpha[AI]$$

20

$$\frac{d[II]}{dt} = -2\mu[II] + 2(1-p)\kappa[LI] - 2\alpha[II]$$

$$\frac{d[IR]}{dt} = -2\mu[IR] - \alpha[IR] + \eta[IA] + 2\alpha[II] + (1-p)\kappa[LR]$$

$$\frac{d[RR]}{dt} = -2\mu[RR] + 2\alpha[IR] + 2\eta[AR]$$

Pair Approximation Graph

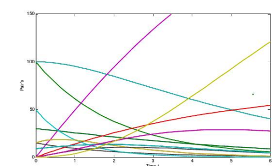


Figure 10: Close up for Pair Approximation Deterministic

This graph is the deterministic model for the equations for the pair approximation. Each value has an initial condition and parameters.

Conclusion

Random graphs allow us to simulate experiments in which we can monitor the response of a disease on a population. This is a very helpful tool that can aid the help in the healthcare of the population and also it gives us a basic understanding of the true potential any disease can impact society

References

- •McSweeney, John wrote all R code that created the graphs.
- •McSweeney, John. "Random Epidemic Spread on Structured Populations." Samsi. 26 Feb. 2010. Lecture.
- •Newman, M.E.J. "The Structure and Function of Complex Networks." *SIAM Review* 45.2 (2003): 167-256. Web.