4.

$$U_E = \frac{1}{2}\frac{Q_m^2}{C}$$

$$1.40\times10^{-6} = \frac{1}{2}\frac{\left(1.6\times10^{-6}\right)^2}{C}$$

$$C = \frac{\left(1.6\times10^{-6}\right)^2}{2\left(1.40\times10^{-6}\right)} = 9.14\times10^{-9}$$

5.

$$\frac{1}{2}\frac{Q_m^2}{C} = \frac{1}{2}Li_m^2$$

$$\frac{\left(3\times10^{-6}\right)^2}{4\times10^{-6}} = \left(1.1\times10^{-3}\right)i_m^2$$

$$2.25\times10^{-6} = \left(1.1\times10^{-3}\right)i_m^2$$

$$i_m^2 = \frac{2.25\times10^{-6}}{\left(1.1\times10^{-3}\right)} = 2.05\times10^{-3}$$

$$i_m = 45.2\,\text{mA}$$

11.

$$\text{a)}f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$\frac{f_{max}}{f_{min}} = \frac{\left(LC_{min}\right)^{\frac{1}{2}}}{\left(LC_{max}\right)^{\frac{1}{2}}} = \sqrt{\frac{C_{max}}{C_{min}}}$$

$$\frac{f_{max}}{f_{min}} = \sqrt{\frac{365}{10}} = 6.04 \text{ or } 6:1$$

$$\text{b)}\frac{1.6}{.54} = \sqrt{\frac{365+C}{10+C}}$$

$$C = 35.63\text{pF}$$

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\left[L(C+10)\right]^{\frac{1}{2}} = 1.6\text{MHz}$$

$$L = 2.2\times10^{\ 4}\text{H}$$

13.

$$a)\ Q = Q_m \sin(\omega t)$$

$$I = \frac{dQ}{dt} = \omega Q_m \cos(\omega t)$$

$$I_m = \omega Q_m$$

$$Q_m = \frac{I_m}{\omega} = I_m \sqrt{LC}$$

$$Q_m = 2\left[\left(3\times10^{-3}\right)\left(2.7\times10^{-6}\right)\right]^{\frac{1}{2}}$$

$$Q_m = 1.8\times10^{-4}\ \text{coulombs}$$

$$b)\ U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C} Q_m^2 \sin^2(\omega t)$$

$$\frac{dU}{dt} = \frac{1}{2C} Q_m^2 \left[2\sin(\omega t)(\omega\cos(\omega t))\right]$$

Use $2\sin x\cos x = \sin 2x$

$$\frac{dU}{dt} = \frac{\omega Q_m^2}{2C}\sin(2\omega t)$$

Greatest rate occurs when $\sin(2\omega t) = 1.$

$$\text{So } 2\omega t = \frac{\pi}{2} \text{ or } 2\left(\frac{2\pi}{T}\right)t = 2\pi$$

$$t = \frac{T}{8}$$

c) in eq. $\frac{dU}{dt} = \frac{\omega Q_m^2}{2C}\sin(2\omega t)$

$$\omega = \frac{2\pi}{T} \text{ and } \sin(2\omega t) = 1$$

$$\left.\frac{dU}{dt}\right)_{max} = \frac{\pi Q^2}{TC}$$

$$T = 2\pi\sqrt{LC} = 5.7\times10^{-4}\ \text{sec}$$

$$\left.\frac{dU}{dt}\right)_{max} = 66.7\ \text{watts}$$

29.

a) $V_L = V_{max}\sin(\omega t)$

$$V_L = L\frac{di}{dt} \Rightarrow \frac{di}{dt} = \frac{V_L}{L} = \frac{V_{max}\sin(\omega t)}{L}$$

$$\int \frac{di}{dt}dt = \int \frac{V_{max}\sin(\omega t)}{L}dt$$

$$i = -\frac{V_{max}}{\omega L}\cos(\omega t) = -\frac{30}{(2\pi \times 10^3)(50 \times 10^{-3})}\cos(\omega t)$$

$i = .095\,\text{amps}$

b) $$i = -\frac{30}{(2\pi)(8 \times 10^3)(50 \times 10^{-3})}\cos(\omega t)$$

$i = .0119\,\text{amps}$

31.

a) $X_L = \omega L,\ X_C = \frac{1}{\omega C}$

$$\omega(6 \times 10^{-3}) = \frac{1}{\omega(10 \times 10^{-6})}$$

$$\omega^2 = \frac{1}{60 \times 10^{-9}}$$

$\omega = 4.1 \times 10^3$

$f = .65\,\text{kHz}$

b) $X_L = \omega L = (4.1 \times 10^3)(6 \times 10^{-3})$

$X_L = 24.6\,\Omega$

c) $$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{6 \times 10^{-8}}} = .65\,\text{kHz}$$

Same as part a).

32.

a) $X_L = \omega L = 377(12.7) = 4790\,\Omega$

$$i_m = \frac{25}{4790} = .0052\,\text{amps}$$

b) Must be 90° out of phase or 0 volts.

c) $\mathcal{E} = -12.5$ @ $t = ?$

$$25\sin(\omega t) = -12.5$$

$$\sin(\omega t) = \frac{1}{2} \Rightarrow \omega t = 30^\circ \text{ or } \frac{\pi}{6}$$

Since current must be out of phase with voltage.

$$I = I_m \cos\left(\frac{\pi}{6}\right) = .0052(.866) = 4.5\,\text{mA}$$

39. From sample problem 33-5

$C = 15\mu F$

$L = 230H$

$f = 60Hz$

$\mathcal{E}_m = 36V$

$R = 160\Omega$

remove inductor

$$Z = \sqrt{R^2 + (X_c)^2}$$

$$X_c = \frac{1}{\omega c} = \frac{1}{377(15 \times 10^{-6})} = 177\Omega$$

$$Z = \sqrt{160^2 + 177^2} = 238.6\Omega$$

$$I = \frac{\mathcal{E}}{Z} = \frac{36}{238.6} = .15\text{amps}$$

$$\tan\phi = \frac{-X_c}{R} = \frac{-177}{160} = -1.11$$

$$\phi = -47.9^\circ$$

47.

$$a)\, i_m = \frac{E_m}{\sqrt{R^2 + \left(\omega L \quad \frac{1}{\omega C}\right)^2}}$$

When is $\frac{di_m}{d\omega} = 0$?

$$\frac{di_m}{d\omega} = \quad \frac{E_m}{2}\left[R^2 + \left(\omega L \quad \frac{1}{\omega C}\right)^2\right]^{\frac{3}{2}}\left[2\left(\omega L \quad \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)\right]$$

$$\frac{di_m}{d\omega} = 0 \text{ when } \omega L \quad \frac{1}{\omega C} = 0$$

$$\omega = \frac{1}{\sqrt{LC}} = \omega_0$$

$$\omega_0 = \left[1\left(20 \times 10^{\ 6}\right)\right]^{\frac{1}{2}} = 223.6 \frac{rad}{sec}$$

$$b)\, \text{max value } i = \frac{E_m}{R} = \frac{30}{5} = 6 amps$$

$$c)\, \frac{1}{2}\frac{E_m}{R} = \frac{E_m}{\sqrt{R^2 + \left(\omega L \quad \frac{1}{\omega C}\right)^2}}$$

$$(2R)^2 = R^2 + \left(\omega L \quad \frac{1}{\omega C}\right)^2$$

$$3R^2 = \omega^2 L^2 \quad \frac{2L}{C} + \frac{1}{\omega^2 C^2}$$

Multiply thru by $\omega^2 C^2$ and collect like terms.

$$\left(L^2 C^2\right)\omega^4 \quad \left(3R^2 C^2 + 2LC\right)\omega^2 + 1 = 0$$

$$\left(4 \times 10^{\ 10}\right)\omega^4 \quad \left(3 \times 10^{\ 8} + 4 \times 10^{\ 5}\right)\omega^2 + 1 = 0$$

$$\left(4 \times 10^{\ 5}\right)\omega^4 \quad 4.003\omega^2 + 10^5 = 0$$

Let $x = \omega^2$, then use the quadratic formula and positive ω's.

$$x = 5.197 \times 10^4, 4.81 \times 10^4$$

$$\omega_1 = 227.98 \frac{rad}{sec} \omega_2 = 219.32 \frac{rad}{sec}$$

$$d)\, \frac{\omega_1 \quad \omega_2}{\omega_0} = .0387$$

61.

a) $i = i_m \sin(\omega t - \varphi)$

$\cos\varphi = \cos(-42^\circ) = .743$

b) Since $\varphi < 0$, i_m leads E.

c) $\tan(-42^\circ) = -.9 = \frac{X_L - X_C}{R}$

So $X_C > X_L$.

d) At resonance, $X_C = X_L \Rightarrow \tan\varphi = 0$.

The circuit is not at resonance.

e) LR No

CR Possible

LCR Possible

f) $P_{AVE} = \frac{1}{2} E_m i_m \cos\varphi$

$P_{AVE} = (.5)(75)(1.2)(.743) = 33.4 \text{watts}$

g) The answers depend on the frequency used in the phase constant φ which is given.

66.

sample problem 7

$$r_t = \left(\frac{N_1}{N_2}\right)^2 R$$

power delivered to R

$$P_{ave} = i_2^2 R = \left(\frac{N_1}{N_2} i_1\right)^2 R$$

using $i = \frac{\mathcal{E}}{R}$ **and since total rest of amps is** $r + rt$

$$P_{ave} = \left(\frac{N_1}{N_2}\right)^2 \frac{\mathcal{E}_m^2 \sin^2 \omega t}{\left(r + \left(\frac{N_1}{N_2}\right)^2 R\right)^2} R$$

let $x = \left(\frac{N_1}{N_2}\right)^2$

$$P_{ave} = \frac{x}{(r + xR)^2} = x(r + xR)^{-2}$$

$$\frac{dP}{dx} = \frac{(r - xR)}{(r + xR)^3} = 0$$

true if $x = \frac{r}{R} = \left(\frac{N_1}{N_2}\right)^2 = \frac{1000\Omega}{10\Omega}$

$$\frac{N_1}{N_2} = 10$$

$$\frac{d}{dx}(x(r + xR)^{-2}) = (r + xR)^{-2} - 2x(r + xR)^{-3} R$$

$$= \frac{r + xR - 2xR}{(r + xR)^3} = \frac{r - xR}{(r + xR)^3}$$

Additional Problem

66.

$$P_{AVE} = \left(I_{RMS}^2\right)(R)$$

$$I_{RMS} = \left(\frac{\mathcal{E}_{RMS}}{Z}\right)$$

$$P_{AVE} = \left(\frac{\mathcal{E}_{RMS}}{Z}\right)^2 R$$

For a pure resistive curcuit, Z = R.

$$P_{AVE} = \frac{\mathcal{E}_{RMS}^2}{R}, \text{ O.K.}$$

Same for a RLC resonant circuit.

In a pure capacitive circuit, R = 0.

$$P_{AVE} = \left(\frac{\mathcal{E}_{RMS}}{Z}\right)^2 R = 0$$

$$\therefore P = 0 \quad \text{O.K.}$$