Chapter 2
1-Dimensional Motion
Quantities for motion

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<td>displacement</td>
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<td>Speed</td>
<td>velocity</td>
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<td>acceleration</td>
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Remember that scalar quantities only have magnitude.
Definitions

• Distance = How far apart two points in space are from each other. (only has magnitude)

• Displacement = The relative location of one point in space to another point. (involves direction)

• Speed = How fast an object is traveling.

• Velocity = How fast and in what direction an object is traveling.

• Acceleration = The changing of an object’s velocity with time. (This can include the velocity’s direction.)

• Examples: Speeding your car up.
  Keeping your speed constant but driving around a curve.
Coordinate systems

When talking about directions, we need to have a reference frame. (coordinate system)

When setting up a coordinate system you can pick any direction to be positive.

For this coordinate system, $x$ and $y$ are positive in the right and up directions respectively.
Example of distance and displacement

The distance between A and B is 5 m.

The displacement of B with respect to A is 5 m to the right of A.

You could say that point B is 5 m east of point A.
Speed and velocity

• Speed

\[ S \] How fast an object is moving at that instant.
\[ S_{ave} = \frac{\text{total distance traveled}}{\text{total time}} \]

• Velocity

\[ V \] How fast and in what direction an object is moving.
\[ V_{ave} = \frac{\text{total displacement}}{\text{total time}} \]

\[ v_{ave} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \]
Velocity

\[ v_{ave} = \frac{\Delta x}{\Delta t} = \frac{18 \text{ m} - 3 \text{ m}}{12 \text{ s} - 7 \text{ s}} = 3 \frac{\text{ m}}{\text{ s}} \]

Direction is to the right.
• For constant velocity: \( V_{\text{ave}} \) is just \( V \)

\[
\nu_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{x_f-x_i}{t_f-t_i} \quad \text{becomes} \quad V = \frac{x_f-x_i}{t}
\]

Rearranging the equation gives:

\[
x = x_0 + vt
\]

for constant velocity

(Just notation, most of the time we will rename \( x_f \Rightarrow x \) and \( x_i \Rightarrow x_0 \))

\( x \) is the value of the position at time \( t \).

\( x \) (position) as a function of \( t \) (time)
Example

Estimate the average speed of an Apollo spacecraft in m/s if it took 5 days to reach the moon. The moon is $3.8 \times 10^8$ m from the Earth.
Calculate the average speed of an Apollo spacecraft in m/s if it took 5 days to reach the moon. The moon is $3.8 \times 10^8$ m from the Earth.

1 day is $60 \times 60 \times 24$ seconds or $86400$ s

So 5 days is $432000$ s

Average Speed = distance/time

$= \frac{3.8 \times 10^8 \text{ m}}{4.32 \times 10^5 \text{ s}}$

$= 880$ m/s
Graphical Interpretation of velocity

If you have a graph of position versus time, the slope of the graph between any two points is the average velocity in that time interval.

As the two points on the graph approach each other, the slope between them becomes the instantaneous velocity at that time.
If you have a graph of position versus time, the slope of the line connecting two endpoints represents velocity.
As the time separation is changed, the slope of the blue lines change. As the time interval approaches zero, the slope of the tangent line represents the instantaneous velocity.

Draw this better on the board.
Instantaneous Velocity

Determined by the slope of a line tangent to the position vs. time curve, at that instant.

Not very steep, lower velocity

Steepest, highest velocity
Graphical Representation of Velocity

If you have a graph of velocity versus time.

The area under the curve represents the net displacement.

Positive area above the horizontal axis represents positive displacement.
Negative area below the horizontal axis represents negative displacement.
Usain Bolt owns the world record in the 100 meters with a time of 9.58 seconds.

Quickly estimate his average speed in m/s. (round the time to 10s)

What was his average speed?
Baseball player hits a home run

The batter stands at home plate, hits a homerun and runs around the bases. The distance between each base is 90 feet.

After returning to home plate...

a) What is the player’s total distance traveled?
b) His displacement?
c) Average velocity? (Hint, the time involved doesn’t matter.)
Drag Race

Watching a drag race from overhead you see two cars taking two different paths from the same starting and end points. If they arrive at the end point at the same time:

a) Do they have the same average velocity?

b) Do they have the same average speed?
Acceleration

Defined as the time rate of change of the velocity.

Acceleration is a vector quantity.

Acceleration can include the velocity’s magnitude changing, the direction of the velocity changing, or both.

\[
a_{\text{ave}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}
\]
Acceleration

Again, as the time separation is changed, the slope of the blue line changes. As the time interval approaches zero, the slope of the tangent line represents the instantaneous acceleration.
Instantaneous Acceleration

Not very steep, Velocity changes slowly

Steepest, velocity changes rapidly
Match the velocity vs. time graph with the corresponding acceleration vs. time graph.
Match the velocity vs. time graph with the corresponding acceleration vs. time graph.

Answers: a-2  
         b-1  
         c-3
1-Dimensional motion with **Constant Acceleration**

\[ a_{ave} = \frac{\Delta v}{\Delta t} = \frac{v_f-v_i}{t_f-t_i} \] becomes \[ a = \frac{(v_f-v_i)}{t} \]

Rewrite this as \[ v = v_0 + at \]

If \( a \) is constant, then \[ v_{ave} = \frac{v_0+v}{2} \]

Since \( \Delta x = v_{ave}t \) then \[ \Delta x = \frac{1}{2} (v_0+v)t \]

Combining: \( \Delta x= \frac{1}{2} (v_0+v)t \) and \( v = v_0 + at \)

We get: \( \Delta x= \frac{1}{2} (v_0 + v_0 + at)t \) or \[ \Delta x= v_0t + \frac{1}{2} at^2 \]
On page 39, by doing some more substituting, we can derive one more equation:

Using: $\Delta x = \frac{1}{2} (v_0 + v)t$ and $t = \frac{(v - v_0)}{a}$

We obtain: $v^2 = v_0^2 + 2a\Delta x$
Three very important equations (found in table 2.4)

For **constant acceleration**

Velocity as function of time.

\[ v = v_0 + at \]

Displacement as a function of time.

\[ \Delta x = v_0 t + \frac{1}{2} at^2 \]

Velocity as a function of displacement. Also called **timeless equation**.

\[ v^2 = v_0^2 + 2a\Delta x \]
Example 2.6 Runway length.

• A plane lands with a speed of 160 mph and can decelerate at a rate of 10 mi/hr/s. If the plane moves with constant speed of 160 mph for 1.0 s after landing before applying the brakes, what is the total runway length needed to come to rest?
Example 2.6 Runway length.

A plane lands with a speed of 160 mph and can decelerate at a rate of 10 mi/hr/s. If the plane moves with constant speed of 160 mph for 1.0 s after landing before applying the brakes, what is the total runway length needed to come to rest?

First convert all quantities to SI units.

$v_0 = 160 \text{ mph} (1 \text{ mph} = 0.447 \text{ m/s}) \Rightarrow 71.5 \text{ m/s}$

$a = -10 \text{ mi/hr/s} = -4.47 \text{ m/s}^2$ (Negative because plane slows down in direction of motion.)
For the 1.0 s the plane travels before applying brakes:
During this time the plane doesn’t slow down. \( a = 0 \)
\[
\Delta x_{\text{coasting}} = v_0 t + \frac{1}{2} a t^2 = (71.5 \text{ m/s})(1.0 \text{ s}) + \frac{1}{2} (0)(1.0 \text{ s})^2 = 71.5 \text{ m}
\]

While braking to a stop:
Use \( v^2 = v_0^2 + 2a\Delta x \)
\[
\Delta x_{\text{braking}} = \frac{(v^2 - v_0^2)}{2a} = \frac{0 - \left(\frac{71.5 \text{ m}}{s}\right)^2}{2 \left(-\frac{4.47 \text{ m}}{s^2}\right)} = 572 \text{ m}
\]

Total length of runway is 71.5 m + 572 m = 644 m
Free Falling Bodies

For free falling objects the acceleration is due to gravity; \( a = g = 9.8 \text{ m/s}^2 \) in the downward direction. (If the coordinate system states that up is positive, then gravity is negative.)

For close estimates you can use \( g = 10 \text{ m/s}^2 \)

Suppose you dropped a ball from a high cliff. After each second interval, the magnitude of the ball’s velocity increases by about 10 m/s.
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</tr>
<tr>
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<td>10</td>
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<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
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(Approximate velocities)
Notice that in each 1 second time interval, the distance traveled increases.

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Remember the equation: \[ \Delta x = v_0t + \frac{1}{2}at^2 \]
The displacement as a function of time is quadratic.
Free falling object

If acceleration is constant, a graph of position vs. time will look like this:
Ballistic Rocket
(note: I changed the numbers from ex 2.10 )

A rocket moves straight upward, starting from rest with an acceleration of +40 m/s\(^2\). It runs out of fuel after 8 seconds and continues upward to a maximum height before falling back to Earth.

a)Find the rocket’s velocity and position after 8 s.
b)Find the maximum height the rocket reaches.
c)Find the velocity the instant the rocket hits the ground.
What we know:

Rocket starts from rest. $v_0 = 0$
acceleration = 40 m/s$^2$ upward

time of acceleration is 8 s.

After 8 seconds, $a = g$ (downward)

A) For the first 8 seconds, the rocket accelerates, use:

$v = v_0 + at$ and $\Delta x = v_0t + \frac{1}{2} at^2$
After 8 seconds:
\[ v = v_0 + at = 0 \text{m/s} + (40 \text{ m/s}^2)(8 \text{ s}) = 320 \text{ m/s} \]
\[ \Delta x = v_0t + \frac{1}{2}at^2 = (0 \text{m/s})(8 \text{ s}) + \frac{1}{2}(40 \text{m/s}^2)(8 \text{s})^2 \quad \Delta x = 1280 \text{ m} \]

B) Next the rocket slows down until it reaches the maximum height.

Now: \[ v_0 = +320 \text{ m/s}, \quad a = g = -9.8 \text{ m/s}^2 \]
We also know that at the top, the velocity is zero. \[ v_f = 0 \text{m/s}. \]

Use : \[ v_f^2 = v_0^2 + 2a\Delta x \] to find \( \Delta x \)
\[ \Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{0 - \left(\frac{320 \text{m}}{s}\right)^2}{2(-9.8 \frac{\text{m}}{s^2})} = 5224 \text{m} \]
In first 8 seconds the rocket traveled 1280 m.
After running out of fuel the rocket traveled another 5224 m.
Total height is 6504 m.

Notice in example 2.10, the book used a different method.
They both work.

C) Find the rocket’s velocity just as it hits the ground.
   At peak of trajectory, v = 0
   Rocket drops 6504 m (displacement is negative)
a = g (downward)
Use the equation: \( v_f^2 = v_0^2 + 2a\Delta x \)

\[
v^2 = (0 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(-6504 \text{ m}) = 127478 \text{ (m/s)}^2
\]

Taking the square root gives us \( v = 357 \text{ m/s} \).

What about the sign?? The velocity when it hits the ground is downward. All the other downward quantities in this problem were negative. The final velocity will be negative. (-357 m/s).