Ch 5
Work and Energy

- Energy
  - Provide a different (scalar) approach to solving some physics problems.
- Work
  - Links the energy approach to the force (Newton’s Laws) approach.
- Mechanical energy
  - Kinetic energy – related to motion
  - Potential energy – related to position

Work

Basic definition of work: $W = F\, d$ (force)(displacement)

Only gives the magnitude of the work **if the force is constant and directed parallel to the displacement**.

Better definition $W = F_x \Delta x$

Force in the direction of the displacement times the displacement.

SI units Joule (J) = Newton meter = $kg\, m^2/s^2$

Work is a scalar.

Work

$W = \mathbf{F}\Delta \mathbf{x}$

Work depends on the displacement.

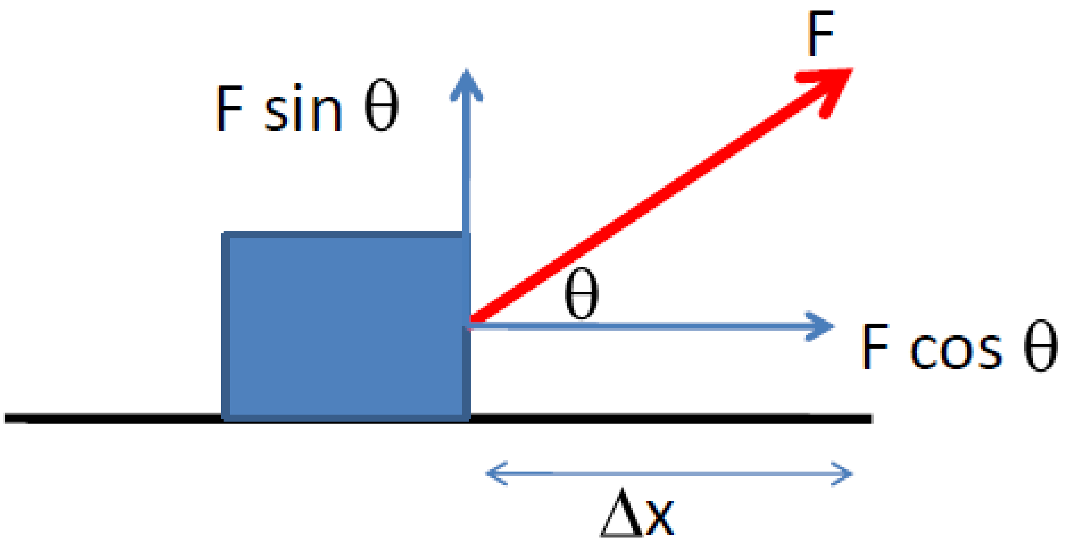
If there is no displacement, there is no work done.

If you are holding a book still in the air, you are not doing work.

If you are sitting in a chair, gravity is not doing any work.

If you are pushing with everything you have against an immoveable object, you are not doing any work.

Only components of forces parallel to the displacement do work.



$$W = (F \cos \theta)\Delta x$$

The vertical component does no work.

$W = (F \cos \theta)\Delta x$

In this definition:

F and Δx are magnitudes, are always positive.

The work will be positive or negative based on the $\cos \theta$.

This formula of work is an example of a “dot product” of vectors.

Work has no direction, but the directions of the force and the displacement matter.

Example

A 500 N force directed 30 degrees above the horizontal is used to pull a 50 kg sled across the ground. The coefficient of friction between the sled and the ground is 0.3. The sled is pulled 5 meters.

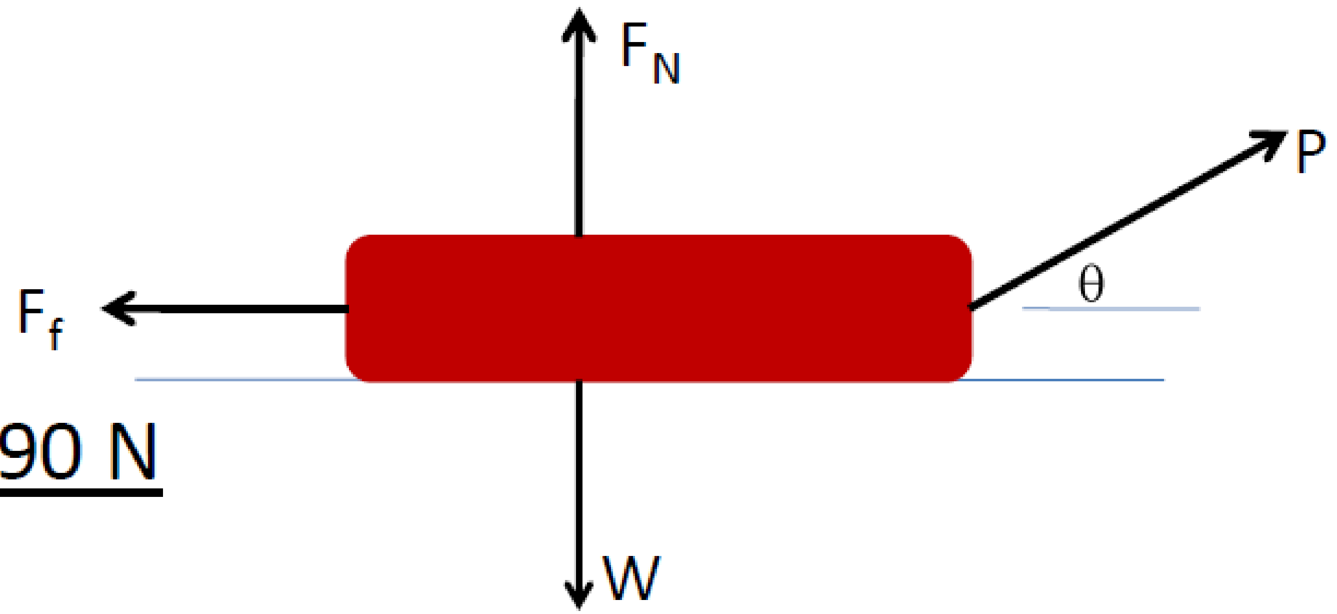
A) What is the work done by the pulling force?
B) Work done by friction?
C) Work done by the normal force?
D) Work done by gravity?

$P = 500\ N, \quad \theta = 30^0$

$m = 50\ kg$

$\mu = 0.3$

$\Delta x = 5\ m$



$W = mg = \underline{490\ N}$

$F_N + Py = W$

$F_N = W - P_y = mg - 500\ N\ (\sin 30)$

$F_N = (50\ kg)g - 250\ N = \underline{240\ N}$

$F_f = \mu F_N = 0.3 * 240\ N = \underline{72\ N}$

$$W = (F \cos \theta)\Delta x$$

Work by pulling force:

W_p = (500 N cos 30) 5 m = 2165 J

Work by friction:

W_f = (72 N cos 180) 5 m = - 360 J

Work by normal force:

W_N = (240 N cos 90) 5 m = 0 J

Work by gravity:

W_g = (490 N cos 270) 5 m = 0 J

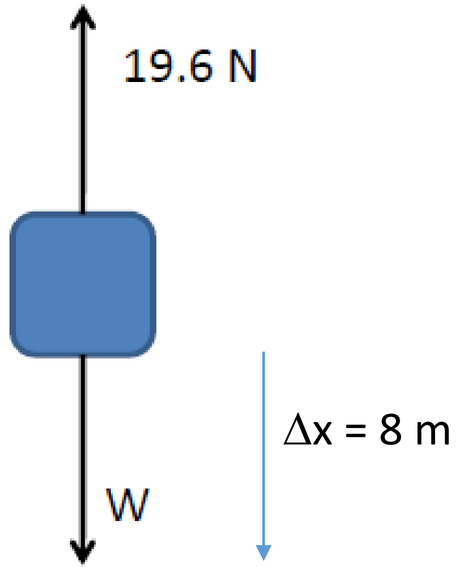
Example

You lower a bucket down a well, with constant velocity a distance of 8 meters. You exert a constant force of 19.6 N. The bucket has a mass of 2 kg. Find the work done by you, and the work done by gravity.

W = mg

W = (2kg) g

W = 19.6 N



The displacement is down the well.

$W = (F \cos \theta)\Delta x$

Your work: $W_y = (19.6\text{ N} \cos 180)\ 8\text{m}$

$W_y = -\ 156.8\text{ J}$

Work by gravity: $W_g = (19.6\text{ N} \cos 0)\ 8\text{m}$

$W_g = 156.8\text{ J}$

Notice that the work done by gravity was mgh, where h is the change in height.

Work-Energy Theorem

- The net work on an object is equal to the object's change in kinetic energy.

- Easy way to find the speed of an object.

- Alternative method to using Newton's Laws.

Kinetic Energy

- Kinetic energy is energy an object has due to its motion.
- Units are Joules
- Kinetic energy is always positive.
- $K = \frac{1}{2} mv^2$

Example. A 1 kg ball is thrown with $v = 20$ m/s

$K = \frac{1}{2}(1kg)(20m/s)^2 = 200$ J

Work-Energy Theorem

Use: $W_{net} = F_{net}\Delta x = (ma)\Delta x$

Also $v^2 = v_0^2 + 2a\Delta x$

Rewrite as: $a\Delta x = (v^2 - v_0^2)/2$

$$W_{net} = m\,(v^2 - v_0^2)/2$$

$$W_{net} = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 \quad \text{or} \quad W_{net} = \Delta K$$

The net work is equal to the change in kinetic energy.

$$W_{net} = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2$$

If the net work is positive, the kinetic energy increases. The speed increases.

If the net work is negative, the kinetic energy decreases. The speed decreases.

Sled example

A 500 N force directed 30 degrees above the horizontal is used to pull a 50 kg sled across the ground. The coefficient of friction between the sled and the ground is 0.3. The sled is pulled 5 meters.

Find the final velocity of the sled.

Assume the sled starts at rest.

Work by pulling force:

W_p = (500 N cos 30) 5 m = 2165 J

Work by friction:

W_f = (72 N cos 180) 5 m = - 360 J

W_{net} = 2165 – 360 J = 1805 J

ΔK = 1805 J

$\Delta K = K_f - K_0$

Since sled starts at rest, K_0 = 0 J

K_f = 1805 J = ½ (50kg)v^2

v^2 = 2(1805 J)/(50g) = 72.2 $(m/s)^2$

<u>v = 8.5 m/s</u>

Bucket example

In this example it was stated that the bucket is lowered at constant velocity.

So $\Delta K = 0$ J Wnet = 0 J

Your work = -156.8 J

$W_g = 156.8$ J

$W_{net} = 0$ J

Conservative and Nonconservative forces

- Conservative forces. IF you do work against a conservative force, that work is recoverable. It can be converted into kinetic energy.

- Examples: gravity, electric force, spring force.

- If you climb a ladder, you did work against gravity. When you jump off and fall to the ground, that work is converted to kinetic energy.

- Conservative forces are path independent. Only the endpoints are important.

Nonconservative Forces

- These forces are dissipative.
- Work done by nonconservative forces cannot be recovered.
- Example: friction
- The work is turned into ‘waste’ energy such as heat or sound.
- They are path dependent.

Potential energy

- Energy that is 'stored up' by doing work against a conservative force.

- Gravitational potential energy.
- Spring potential energy.

- Unit is also the Joule. (J)

Gravitational potential energy

When you raise up an object, you are doing work against gravity. That work is then stored as potential energy. If you drop the object, and let it fall, the potential energy is then transferred to kinetic energy.

$PE = mass*g*\Delta y$ Δy is the change in height

Reference Level/Zero of Potential Energy

- Height that corresponds to zero potential energy.
- If an object is above the line of zero potential energy, it has positive potential energy.
- If an object is below that height, the object has negative potential energy.
- You can set the zero of potential energy line wherever you want. Place it in a location that is convenient to the problem.

Conservation of mechanical energy

<u>Not an equation. This is a rule.</u>

Possibly the most important rule in the universe.

When no dissipative forces are involved, the sum of the kinetic and potential energies is constant.

$$KE_i + PE_i = KE_f + PE_f$$

Energy is converted between the kinetic and potential forms.

Simple example of conservation of energy

A rock is on the edge of a 10 meter high cliff.

Push it off edge of cliff.

You can find the speed that the rock hits the ground.

When the rock is sitting on the cliff, it has no kinetic energy, but it has potential energy.

When the rock is hitting the ground, that potential energy was transferred to kinetic energy.

$\Delta y = 10m$ mass = 50 kg

$KE_i + PE_i = KE_f + PE_f$

$0 + PE_i = KE_f + 0$

$mgh_i = \frac{1}{2} mv^2$

$(50kg)g\ (10m) = \frac{1}{2}\ (50kg)v^2$ $v = 14\ m/s$

Notice the mass canceled out. This is why, neglecting air resistance, all falling bodies accelerate the same because of gravity.

https://www.youtube.com/watch?v=74MUjUj7bp8

A skier is skiing 20 m/s on level ground when the ground begins to slope downward. If the vertical drop is 10 meters, how fast is the skier moving at the bottom of the slope?

$KE_i + PE_i = KE_f + PE_f$

$KE_i + PE_i = KE_f + 0$

$\frac{1}{2}\, m\, (20 m/s)^2 + mg(10 m) = \frac{1}{2}\, mv^2$

$\frac{1}{2}\, (20 m/s)^2 + g(10 m) = \frac{1}{2}\, v^2$

$v = 24.4$ m/s

Springs

Springs can be used to apply forces.

Springs can be used to store energy.

These can be done by either <u>compression</u>, <u>stretching</u>, or torsion (twisting).

Springs

Ideal, or linear, springs follow a rule called Hooke's Law:

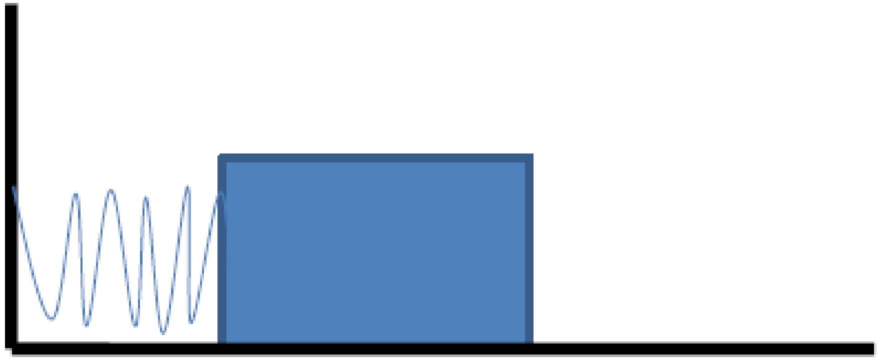
Hooke's Law: $F_s = -k\,x$ Force is 'linear' with x

k is called the spring constant. This determines how stiff the spring is.

x is the distance the spring is deformed (stretched or compressed) **from the equilibrium length**.

The minus sign tells us that this is a restoring force.

Restoring force means that the force the spring exerts, is in the opposite direction of the force that deforms the spring.



If I pull the box to the right (stretch the spring), the spring will exert a force to the left.

If I push the box to the left (compress the spring), the spring will exert a force to the right.

Spring constant

The spring constant, (k), determines how stiff a spring is.

- High spring constant

  Stiff or strong, Hard to stretch or compress

- Low spring constant

  Limp or weak, Easy to stretch or compress

- Units for spring constant are force per length: N/m

Basic Spring Example

A spring with a spring constant of 250 N/m has a length of 0.5 meters when unstretched.

What **magnitude** of force is needed to stretch the spring so that is 0.75 meters long?

$F_s = -k\,x$

$F_s = (250\ \text{N/m})(0.75\text{m} – 0.5\text{m}) = 62.5\ \text{N}$

Note that it would take the same force to compress the spring by 0.25 meters.

Work Done by Springs

$W = F \Delta x$

Because the spring force varies as we stretch/compress the spring, we have to use the average force between the two endpoints.

(This uses the assumption that the force is linear.

$F_{ave} = (F_0 + F_1)/2 = (0 – kx)/2 = -½ kx$

Remember x is the change in the spring's length.

$W_s = -½ kx^2$

See work done by varying forces on page 155. (force varies linearly)

Spring Work

Notice the ‘sign’ of the work.

When you stretch or compress the spring, you do positive work.

The spring does negative work. This is because the spring’s force is opposite to the displacement.

When you let go and the spring returns to the initial condition, the spring then does positive work as the force and displacement are in the same direction.

Elastic Potential Energy

Energy you store up by changing a spring's length is: $PE_s = \frac{1}{2} kx^2$

This is the energy you store when doing work against a spring.

Conservation of Energy

We can involve springs in conservation of energy examples.

The spring energy is a type of potential energy.

See examples 5.10

Conservation of Energy/Work Energy

What do you do when there are nonconservative forces, such as friction?

$$KE_i + PE_i = KE_f + PE_f$$

Becomes

$KE_i + PE_i = KE_f + PE_f +$ Magnitude of work of friction

Or

$$KE_i + PE_i = KE_f + PE_f + W_{nc}$$

$$KE_i + PE_i = KE_f + PE_f + W_{nc}$$

On the left hand side you have the initial mechanical energy.

On the right hand side you have the final mechanical energy PLUS the ***magnitude*** of the 'wasted' energy produced by nonconservative work.

Slide problem.

A 25 kg child climbs a 3 meter high ladder to get onto a slide. The slide has no friction. The length of the slide is 5 meters. Find the speed that the child reaches the ground with.

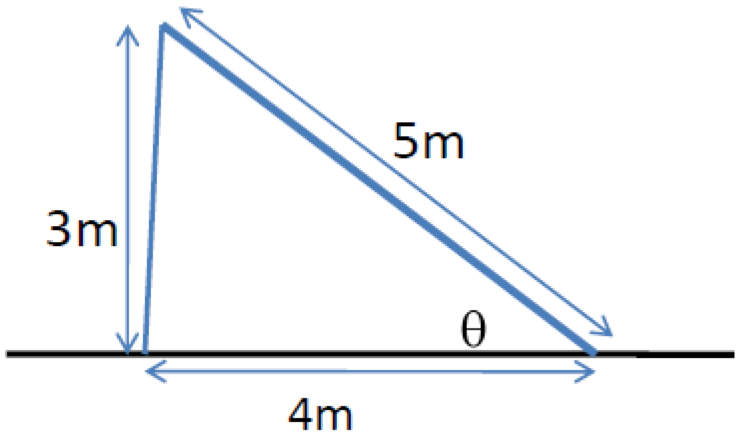
Use CoE (picked ground for zero PE line)

$KE_i + PE_i = KE_f + PE_f$

$0 + mg\Delta y = \frac{1}{2} mv^2 + 0$

$(25kg)g(3m) = \frac{1}{2} (25kg)v^2$

$v = 7.7$ m/s



Slide problem with friction

Repeat previous problem with friction ($\mu_k = 0.5$)

Now use: $KE_i + PE_i = KE_f + PE_f + W_{nc}$

Find frictional force: $\mu F_n = 0.5(25kg)g \cos\theta$

$\cos\theta = 4/5$

$F_f = 98$ N

Magnitude of the Work done by friction is:

$F_f \Delta x = (98 \text{ N})(5\text{m})$

$W_f = 490$ J

Now find the final speed of the child at the end of the slide.

$$KE_i + PE_i = KE_f + PE_f + W_{nc}$$

$$0 \quad + \quad (25kg)g(3m) = ½\,(25kg)v^2 + 0 + 490\ J$$

$$735\ J = ½\,(25kg)v^2 + 490\ J$$

$$735\ J – 490\ J = 245\ J = ½\,(25kg)v^2$$

$$245\ J = ½\,(25kg)v^2$$

$$v = 4.4\ m/s$$

Recall that when there was no friction the final speed was 7.7 m/s.

More slide problem

Now repeat the problem with a longer slide. The height stays the same. But the slide is now 6 meters long. (This will change the angle to 30 degrees.)

Find magnitude of work done by friction.

$F_f = \mu F_n = 0.5(25\text{kg})g\ (\cos 30) = 106\ \text{N}$

Magnitude of frictional work = $F_f \Delta x$

$W_f = (106\ \text{N})\ 6\text{m} = 636\ \text{J}$

$$KE_i + PE_i = KE_f + PE_f + W_{nc}$$

$$0 \quad + \quad (25kg)g(3m) = \tfrac{1}{2}\,(25kg)v^2 + 0 + 636\ J$$

$$735\ J = \tfrac{1}{2}\,(25kg)v^2 + 636J$$

$$735\ J - 636\ J = 99\ J = \tfrac{1}{2}\,(25kg)v^2$$

$$99\ J = \tfrac{1}{2}\,(25kg)v^2$$

$$v = 2.8\ m/s$$

More slide problem

Now repeat the problem with an even longer slide. The height stays the same. But the slide is now 10 meters long. (This will change the angle to 17 degrees.)

Find magnitude of work done by friction.

$F_f = \mu F_n = 0.5(25\text{kg})g\ (\cos 17) = 117\ \text{N}$

Magnitude of frictional work = $F_f \Delta x$

$W_f = (117\ \text{N})\ 10\text{m} = 1170\ \text{J}$

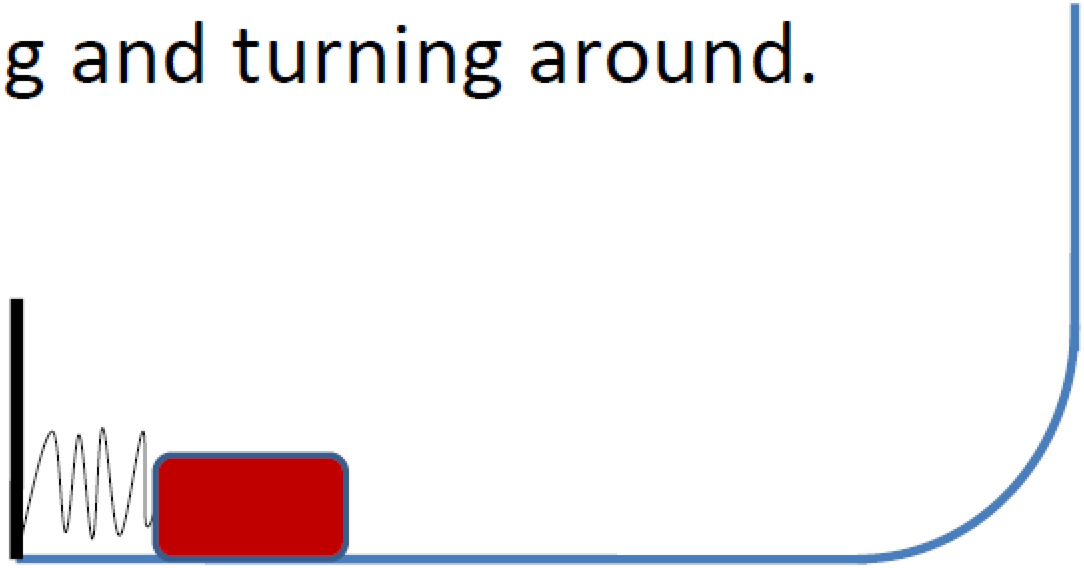
More slide problem

Now the work done by friction is more than the initial potential energy. In this case, the child will not slide all the way down the slide.

The 735 J of potential energy is not enough to overcome all the friction needed to reach the end of the slide.

Cons. of energy problem.

A 2 kg block is compressing a horizontal spring with force constant (k = 25 N/m) by a distance of 2 meters. When the spring releases the block, it follows a path as shown in the picture. Find out high the block will reach before stopping and turning around.



Use Cons. of Energy

I picked the ground level to be the zero of potential energy line.

Originally all the energy is spring potential:

$PE_s = ½\ kx^2$

After the block leaves the spring, it will have kinetic energy:

$KE = ½\ mv^2$

When block reached final height, it will have potential energy:

$mg\Delta y$ You want to find Δy

Initial energy is from the spring:

$PE_s = \frac{1}{2} kx^2 = \frac{1}{2}(25\ N/m)(2m)^2 = 50\ J$

When the block moves on the horizontal segment of the path it has kinetic energy:

$KE = \frac{1}{2} mv^2 = \frac{1}{2}(2kg)v^2 \qquad = 50\ J$

$v = 7.1\ m/s$

At the block's highest point, it has only gravity potential energy:

$mg\Delta y \qquad = 50\ J$

$50\ J = (2\ kg)\ g\ \Delta y$

$\Delta y = 2.6$ meters

You could have skipped the middle step where we found how fast the block leaves the spring.

$PE_s = PE_g$

$\frac{1}{2} kx^2 = mg\Delta y$

$\frac{1}{2} (25 \text{ N/m}) (2\text{m})^2 = (2 \text{ kg})g \, \Delta y$

$\Delta y = 2.6$ meters

Power

Definition of power is the work done per time.

$P = W/\Delta t$

SI unit of power is the watt. [J/s]

U.S. unit of power is horsepower

1 hp = 746 W

For electric power generation/usage, use the kilowatt-hour. This is the energy transferred in 1hr rate the rate of 1kW (1000 J/s).

$1kWh = (1000 J/s)(3600 s) = 3.6 \times 10^6 J$

Remember Work = $F\Delta x$

P = Work/time

$P = F\Delta x/\Delta t$ so: $P = F\,v$ v = velocity

Power is a scalar quantity.

$P_{ave} = F\,v_{ave}$ average power

$P = F\,v$ instantaneous power

Power

- Power is related to how fast a force can be applied.
- Picking up a heavy weight slowly may not require a lot of power.
- Picking up the same weight faster will require more power.
- Weightlifting examples: comparing power required to perform a bench press and a snatch.

https://www.youtube.com/watch?v=jfU2R-PE5rE

https://youtu.be/E_7cOou1ZJk

Similar weights but one of these lifts demonstrates much more power than the other.

I picked some estimated values of weights, distances, and times.

Assume the weights are moved at constant velocities.

Bench press: 300 lbs (1335 N),

Range of motion $\Delta x \sim 0.5$ m

time to raise weight ~3 seconds

$P = F\Delta x/\Delta t$ (1335 N)(0.5 m)/(3s) = 222.5 W

snatch: 100 lbs (445 N)

Range of motion $\Delta x \sim 1.5$ m

time to raise weight ~1 s

$P = F\Delta x/\Delta t$ (445 N)(1.5 m)/(1s) = 667.5 W

Even though the snatch is performed with less weight, it requires more power because of the larger velocity.

Bucket Example

You want to lift a 20 kg bucket up a well at a constant velocity of 0.5 m/s. What power is needed to do so?

Since the velocity is constant, the upward force you must pull with is equal to the weight of the bucket.

$P = F\,v = (20\text{kg})g\,(0.5\text{ m/s}) = 98\text{ W}$

Another bucket example

Again you want to raise the same 20 kg bucket. It is starting from rest, and you want to pull on the bucket so that is has a velocity of 2 m/s. You want to accomplish this over a time interval of 4 seconds.

Use Power and the work energy theorem.

(Net work equals change in kinetic energy.)

First find the change in the kinetic energy.

$\Delta K = ½\ mv_f^2 – ½\ mv_0^2 = ½\ (20kg)(2m/s)^2$

$\Delta K = 40\ J$

Work = 40 J

Power = Work/time = (40 J) /(4s) = 10 W

Shamu example

Calculate the average power needed for the whale to speed up. The killer whale has mass of 8000 kg.

What power is needed to reach speed of 12m/s in a 6 second time interval?

<u>Do work energy theorem</u>.

$\Delta K = ½\, mv_f^2 – ½\, mv_0^2 = ½\, (8000kg)(12m/s)^2$

$\Delta K = 5.76x10^5 J$

Power = $(5.76x10^5 J)/6\ s$ = $9.6x10^4$ W

This was neglecting drag. The power is actually higher.

$9.6x10^4$ W (1hp/746W) = 128 hp

About the same as a car.

Power delivered by elevator motor

A 1000 kg elevator carries a load of 800 kg. A constant friction force of 4000 N retards it upward motion. The retarding force behaves similar to friction. What minimum power in kilowatts and horsepower must the motor deliver to lift the fully loaded elevator at a constant speed of 3 m/s?

Elevator

Since the speed is constant (3m/s), the acceleration and the sum of all forces equals zero.

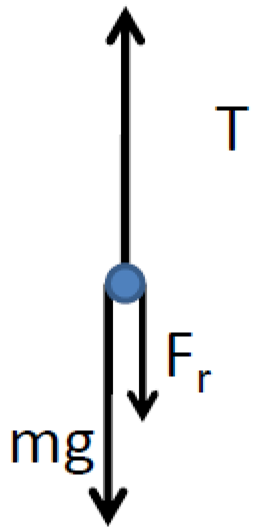
Find the force (T) the motor must pull with to achieve this motion.

$\Sigma F = ma = 0$

$T - F_r - mg = 0$

$T = F_r + mg = 4000\ N + (1800kg)g$

$T = 2.16 \times 10^4 N$



Elevator

Now that we know the force the motor must exert we can find the power using P = Fv.

$P = Fv = (2.16x10^4N)(3m/s) = \underline{6.48x10^4W}$

746 W = 1 hp

So $P = (6.48x10^4W) \times (1\ hp)/(746\ W) = \underline{87\ hp}$

Calculating Power

If the velocity is constant:

Use Power = Force x Velocity

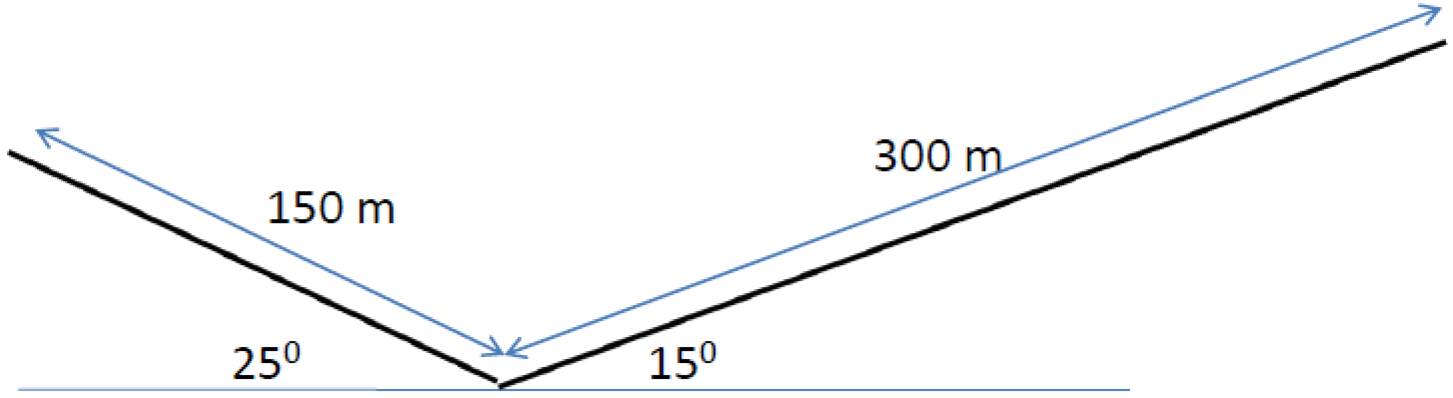
If the velocity is changing:

Use the work energy theorem to find the work.

Then use Power = Work / time

Runaway truck

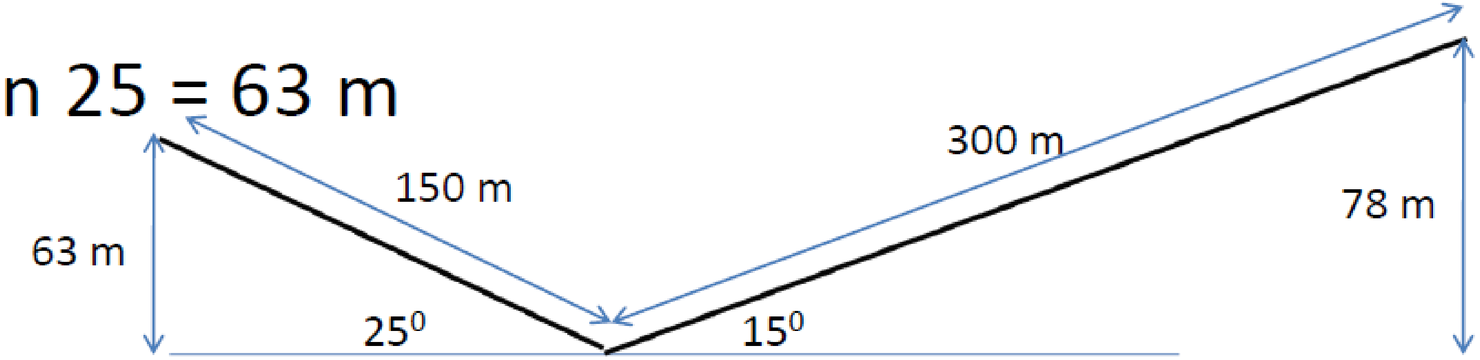
A runaway truck is driving 25 m/s down a hill. The truck has 300 meters to go before reaching the bottom and the hill is on a 15degree angle. At the bottom of the hill is a runaway truck ramp. The ramp is 150 meters long at a 25 degree angle. Will the ramp stop the truck?



First find the important heights.

300 m sin 15 = 78 m

150 m sin 25 = 63 m



Do conservation of energy.

$KE_i + PE_i = KE_f + PE_f$

$\frac{1}{2} m v_i^2 + mgh_i = \frac{1}{2} mv_f^2 + mgh_f$

$\frac{1}{2} m (25m/s)^2 + mg(78m) = mg(63m) + \frac{1}{2} mv_f^2$

$\frac{1}{2} (25m/s)^2 + g(78m) = g(63m) + \frac{1}{2} v_f^2$

v_f = 30.3 m/s   The truck is moving faster.

Runaway truck ramp continued…

Runaway truck ramps will have a rough surface.

(usually in addition they have piles of sand or gravel on them) to produce a lot of friction.

If we include a coefficient of friction of 0.4 between the truck and the ramp, will the ramp stop the truck?

Runaway truck with friction

Use work energy/conservation of energy.

$KE_i + PE_i = KE_f + PE_f + W_{nc}$, where W_{nc} is the work done by friction.

$\frac{1}{2} m v_i^2 + mgh_i = \frac{1}{2} mv_f^2 + mgh_f + \mu_k mg(\cos\theta)d$

$\frac{1}{2} m(25m/s)^2 + mg(78m) = mg(63m) + \frac{1}{2} mv_f^2 + 0.4mg(\cos 25)(150m)$

$\frac{1}{2}(25m/s)^2 + g(78m) = g(63m) + \frac{1}{2}v_f^2 + 0.4g(\cos 25)(150m)$

$313 + 764 = 617 + 533 + \frac{1}{2} v_f^2$

$1077 = 1150 + \frac{1}{2} v_f^2$

v_f^2 can't be negative so the truck was stopped.

We can find how far along the ramp, the truck stops.

$KE_i + PE_i = KE_f + PE_f + W_{nc}$,

$\frac{1}{2} m v_i^2 + mgh_i = \frac{1}{2} mv_f^2 + mgh_f + \mu_k mg(\cos\theta)d$

Change h_f to d*sin 25 since we can relate the final height to the distance along the ramp.

$\frac{1}{2} (25 m/s)^2 + g(78m) = 0 + g\, d^*\sin 25 + 0.4g(\cos 25)d$

$1077 = d\,(g \sin 25 + 0.4\, g \cos 25)$

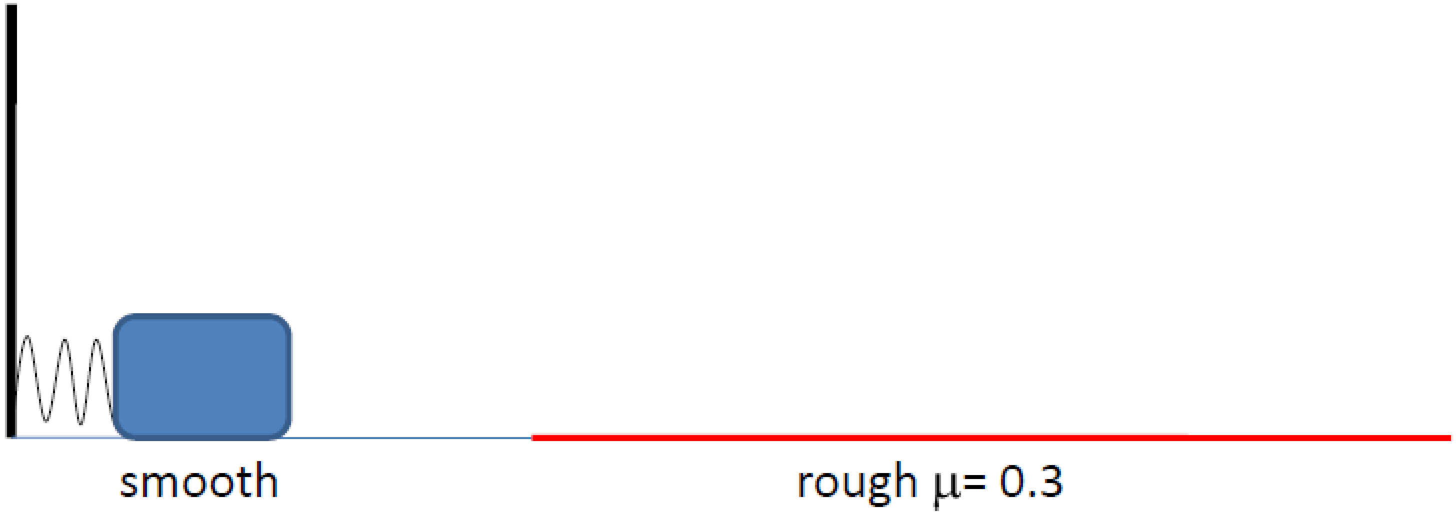
$1077 = d\,(7.7)$

$d = 140$ m

https://www.youtube.com/watch?v=h5KgKebgkwk

https://www.youtube.com/watch?v=qYpIXsbLUgE

A 50 kg block compresses a spring (k = 1000N/m) a distance of 0.5 meters. After the block is released, it moves on a frictionless surface for 2 meters before sliding on a rough surface. If the coefficient of friction is 0.3, how far along on the rough surface, does the block slide?



- Initial situation:

Spring potential energy $PE_s = \frac{1}{2} kx^2$

The block eventually comes to rest. The rough surface does work equal to the initial energy.

remember: $W_f = F_f d = \mu mgd$

$\frac{1}{2} kx^2 = \mu mgd$

$\frac{1}{2}(1000\ \text{N/m})(0.5\ \text{m})^2 = 0.3\ (50\ \text{kg})\ g\ d$

$d = 0.85\ \text{m}$