# Chapter 13. Binomial Distributions

## The Binomial Setting and Binomial Distributions

**Note.** The binomial setting consists of an experiment with observations satisfying:

**1.** There are a fixed number $n$ of observations.

**2.** The $n$ observations are all **independent**. That is, knowing the result of one observation does not change the probabilities we assign to other observations.

**3.** Each observation falls into one of just two categories, which for convenience we call "success" and "failure."

**4.** The probability of a success, call it $p$, is the same for each observation.

**Definition.** The count $X$ of successes in the binomial setting has the **binomial distribution** with parameters $n$ and $p$. The parameter $n$ is the number of observations, and $p$ is the probability of a success on any one observation. The possible values of $X$ are the whole numbers from 0 to $n$.

**Example S.13.1. Binomial Stooges.**
A TV station airs 10 (not necessarily different) stooge films per week. The films are chosen at random from the collection of 190 films. Since there are 97 films with Curly as the third stooge, the probability of a Curly film being chosen at random is $p = 97/190$. Therefore the count of the number of Curly films shown by the TV station follows a binomial distribution with parameters $p = 97/190$ and $n = 10$.

## Binomial Distributions in Statistical Sampling

**Note.** Choose a simple random sample of size $n$ from a population with proportion $p$ of successes. When the population is much larger than the sample, the count $X$ of successes in the sample has approximately the binomial distribution with parameters $n$ and $p$. This property will allow us to answer binomial questions with normal approximations.

## Binomial Probabilities

**Note.** The number of ways of arranging $k$ successes among $n$ observations is given by the **binomial coefficient**

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

for $k = 0, 1, 2, \ldots, n$. The exclamation point indicates **factorial** and for whole number $n > 0$ is $n! = n \times (n-1) \times (n-2) \times \cdots 3 \times 2 \times 1$. By definition, $0! = 1$.

**Note.** If $X$ has the binomial distribution with $n$ observations and probability $p$ of success on each observation, the possible values of $X$ are $0, 1, 2, \ldots, n$. If $k$ is any one of these values,

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}.$$

**Note.** The real reason the term "binomial" is used in this section involves the following idea. Suppose $p$ is a probability and $q = 1 - p$ (i.e., $p + q = 1$). Then by the Binomial Theorem, if we raise $p + q$ to the $n$th power, we get:

$$(p+q)^n = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}.$$

Notice that the $k$th term on the right is the same as the probability that $X = k$.

**Example S.13.2. Binomial Stooges Again.**
In example S.13.1, we considered a binomial experiment with $p = 97/190$ and $n = 10$. Make a table of $P(X = k)$ for $k = 0, 1, 2, \ldots, 10$.

**Solution.** Using the formula

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k},$$

we get:

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P(X = k)$ | 0.0008 | 0.0082 | 0.0386 | 0.1075 | 0.1962 | 0.2455 | 0.2134 | 0.1272 | 0.0498 | 0.0115 | 0.0012 |

## Binomial Mean and Standard Deviation

**Note.** If a count $X$ has the binomial distribution with number of observations $n$ and probability of success $p$, the **mean** and **standard deviation** of $X$ are

$$\mu = np \text{ and } \sigma = \sqrt{np(1-p)}.$$

**Example.** Exercise 13.9 page 334.

# The Normal Approximation to Binomial Distributions

**Note.** Suppose that a count $X$ has the binomial distribution with $n$ observations and success probability $p$. When $n$ is large, the distribution of $X$ is approximately the normal distribution $N(np, \sqrt{np(1-p)})$. As a rule of thumb, we will use the normal approximation when $n$ is so large that $np \geq 10$ and $n(1-p) \geq 10$.

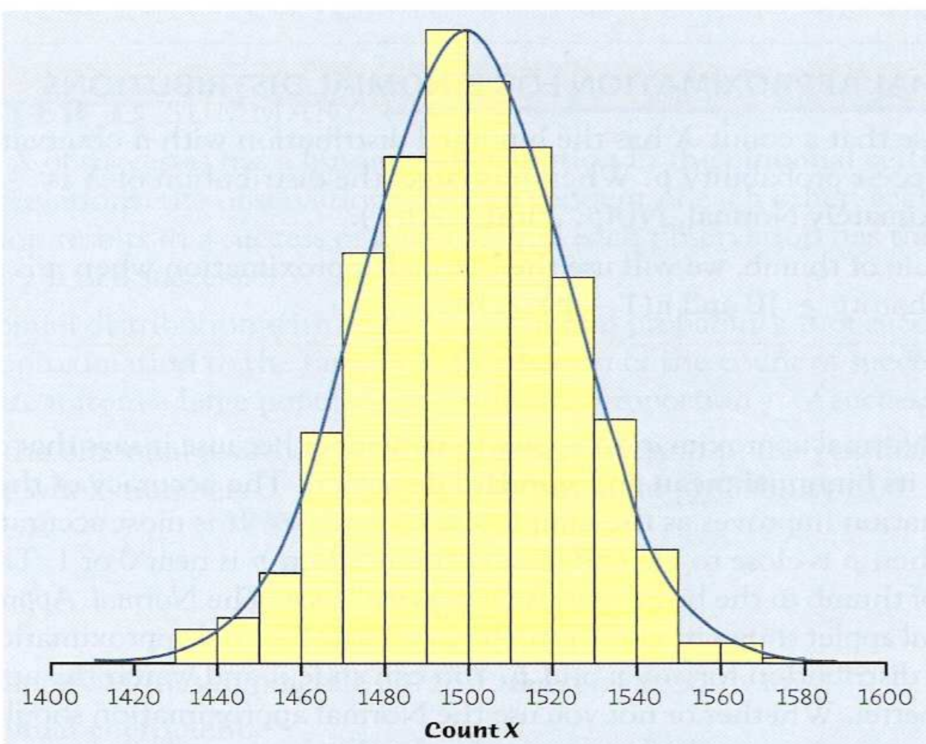


Figure 13.3 page 335. A histogram of 1000 trials of a binomial experiment with $n = 2500$ and $p = 0.6$.

**Example.** Exercise 13.11 page 337

**Example.** Exercise 13.35 page 341.

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