Appendix A. Review.

A.2. Geometry Essentials

Note. In this appendix, we introduce and use the Pythagorean Theorem and its converse, present geometry formulas, and consider congruent and similar triangles.

Definition. A *right triangle* is a triangle that contains a right angle, that is an angle of 90°. The side of the triangle opposite the 90° angle is called the *hypotenuse*; the remaining two sides are called *legs*.

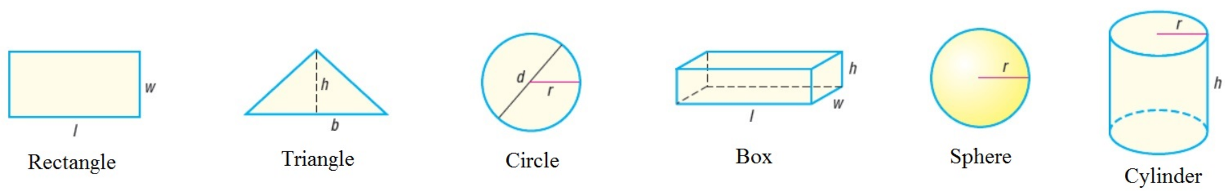
Note. In Euclidean geometry (in which the Parallel Postulate holds) the Pythagorean Theorem holds. Here, we state the Pythagorean Theorem and its converse separately. For more on this, see my brief online presentation A Quick Introduction to Non-Euclidean Geometry or my more detailed presentation Euclidean Geometry.

Theorem A.2.A. The Pythagorean Theorem. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. That is, if the hypotenuse has length c and the two legs have lengths a and b, then $a^2 + b^2 = c^2$.

Theorem A.2.B. The Converse of the Pythagorean Theorem. In a triangle, if the square of the length of one side equals the sum of the squares of the lengths of the other two sides, the triangle is a right triangle. The 90° angle is opposite the longest side.

Examples. Page A19 numbers 14, 26, and page A21 number 56.

Note A.2.A. Consider the following geometric figures.



• For a rectangle of length ℓ and width w, the area is $A = \ell w$ and the perimeter is $P = 2\ell + 2w$

• For a triangle with base b and altitude (height) h, the area is $A = \frac{1}{2}bh$.

• For a circle of radius r (or diameter $d = 2r$), the area is $A = \pi r^2$ and the circumference is $C = 2\pi r = \pi d$.

• For a closed rectangular box of length ℓ, width w, and height h, the volume is $V = \ell wh$ and the surface area is $S = 2\ell h + 2wh + 2\ell w$.

• For a sphere of radius r, the volume is $V = \frac{4}{3}\pi r^3$ and the surface area is $S = 4\pi r^2$. for a closed right circular cylinder of height h and radius r, the volume is $V = \pi r^2 h$ and the surface area if $2\pi r^2 + 2\pi rh$.

Examples. Page A20 numbers 40 and 52.

Definition. Two triangles are *congruent* if each pair of corresponding angles have the same measure and each pair of corresponding sides are the same length.

Note. You likely saw the following in high school geometry.

Theorem A.2.C.

Angle-Side-Angle Case. Two triangles are congruent if two of the angles are equal and the lengths of the corresponding sides between the two angles are equal.

Side-Side-Side Case. Two triangles are congruent if the lengths of the corresponding sides of the triangles are equal.

Side-Angle-Side Case. Two triangles are congruent if the lengths of two corresponding sides are equal and the angles between the two sides are the same.

Definition. Two triangles are *similar* if the corresponding angles are equal and the lengths of the corresponding sides are proportional.

Note. We also have ways of determining if two triangles are similar.

Theorem A.2.D.

Angle-Angle Case. Two triangles are similar if two of the corresponding angles are equal.

Side-Side-Side Case. Two triangles are similar if the lengths of all three sides of each triangle are proportional.

Side-Angle-Side Case. Two triangles are similar if two corresponding sides are proportional and the angles between the two sides are equal.

Examples. Page A20 numbers 44 and 46.

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