Chapter 1. Functions

1.5. Exponential Functions

Note. In Chapter 7 we will use calculus to define logarithmic and exponential functions. In this section and the next, we informally deal with these functions, as you did in highschool.

Definition. If $a \neq 1$ is a positive constant, the function $f(x) = a^x$ is the *exponential function with base a.*

Example. Page 39, numbers 2a and 8a.

Note. If $x = n$ is an integer, then

$$a^x = a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}.$$

If $x = p/q$ is a rational number (a ratio of integers), then $a^x = a^{p/q} = \sqrt[q]{a^p}$. If x is irrational, then the decimal representation of x has no terminating or repeating pattern. We could use rational approximations of x to approximate a^x. For example, $\sqrt{3} \approx 1.732050808$, and we could approximate $2^{\sqrt{3}}$

by using more and more decimals of $\sqrt{3}$:

$$2^1,\ 2^{1.7},\ 2^{1.73},\ 2^{1.732},\ 2^{1.7320},\ 2^{1.73205},\ldots$$

We would then look for the *limit* of the sequence.

Theorem. Rules for Exponents.

If $a > 0$ and $b > 0$, the following rules old true for all real numbers x and y.

1. $a^x \cdot a^y = a^{x+y}$

2. $\dfrac{a^x}{a^y} = a^{x-y}$

3. $(a^x)^y = (a^y)^x = a^{xy}$

4. $a^x \cdot b^x = (ab)^x$

5. $\dfrac{a^x}{b^x} = \left(\dfrac{a}{b}\right)^x$

Example. Page 39, number 12.

Definition. The number e (to be formally defined in Chapter 7) is an irrational number which is approximately

$$e \approx 2.7182818284590459.$$

An exponential function with base e is called the *natural exponential function.*

Note. The exponential functions $y = e^{kx}$, where k is a nonzero constant, are frequently used for modeling exponential growth or decay. The function $y = y_0 e^{kt}$ is a model for exponential growth if $k > 0$ and a model fro exponential decay if $k < 0$.

Examples. Page 40, number 30. Page 39, Example 4.