Chapter 3. Differentiation

3.1 Tangents and the Derivative at a Point

Note. We now return to an idea introduced in Section 2.1: Slopes of lines tangent to curves.

Definition. Slope and Tangent Line.

The *slope of the curve* $y = f(x)$ at the point $P(x_0, f(x_0))$ is the number

$$m = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h},$$

provided the limit exists. The *tangent line* to the curve at P is the line through P with this slope.

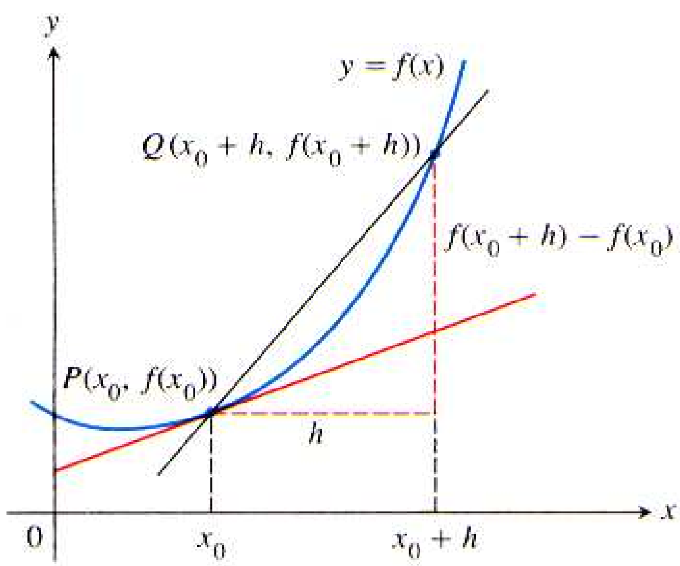

Figure 3.1, page 122

Example. Page 125 number 7.

Definition. Derivative at a Point.

The *derivative of a function f at a point x_0*, denoted $f'(x_0)$, is

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

provided this limit exists.

Example. Page 125 number 26.

Note. Since the derivative of a function at a point is a limit of an average rate of change (to recall a topic from Section 2.1), then we see that the derivative can be interpreted as an instantaneous rate of change of the function f with respect to the variable x. For example, if $f(t)$ is the position of a particle at time t, then the instantaneous rate of change of position with respect to time (i.e. the *instantaneous velocity*) at time $t = t_0$ is

$$f'(t_0) = \lim_{h\to 0} \frac{f(t_0+h) - f(t_0)}{h},$$

provided the limit exists.

Examples. Page 125 number 28 and Page 126 number 36 (vertical tangents).