Chapter 3. Differentiation

3.4 The Derivative as a Rate of Change

Definition. Instantaneous Rate of Change

The *instantaneous rate of change* of f with respect to x at x_0 is the derivative

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h},$$

provided the limit exists.

Definition. (Instantaneous) Velocity

Velocity (*instantaneous velocity*) is the derivative of position with respect to time. If a body's position at time t is $s = f(t)$, then the body's velocity at time t is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

Definition. Speed

Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left|\frac{ds}{dt}\right|$$

Definition. Acceleration, Jerk

Acceleration is the derivative of velocity with respect to time. If a body's position at time t is $s = f(t)$, then the body's acceleration at time t is

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

Jerk is the derivative of acceleration with respect to time:

$$j(t) = \frac{da}{dt} = \frac{d^3s}{dt^3}.$$

Example. Page 152 number 12.

Note. At the surface of the Earth, if an object is fired directly upward with an initial (upward) velocity v_0 from an initial height s_0, then the height of the object at time t is

$$s(t) = -16t^2 + v_0 t + s_0$$

if time is measured in seconds and distances are measured in feet, or

$$s(t) = -4.9t^2 + v_0 t + s_0$$

if time is measured in seconds and distances are measured in meters. Notice what this implies that the accelerations are.

Example. Page 154 number 22.

Note. In economics, the term "*marginal*" is used when referring to derivatives. If a company produces and sells a number x of objects, and the cost of producing those objects is $c(x)$ and the revenue that results from selling them is $r(x)$, then the resulting profit is $p(x) = r(x) - c(x)$. The functions $p'(x)$, $r'(x)$, and $c'(x)$ are the marginal profit, revenue, and cost functions, respectively.

Example. Page 154 number 24.