Chapter 3. Differentiation

3.10 Linearization and Differentials

Definition. If f is differentiable at $x = a$, then the approximating function

$$L(x) = f(a) + f'(a)(x - a)$$

is the *linearization* of f at a.

Example. Page 250 number 2.

Definition. Let $y = f(x)$ be a differentiable function. The *differential* dx is an independent variable. The *differential* dy is

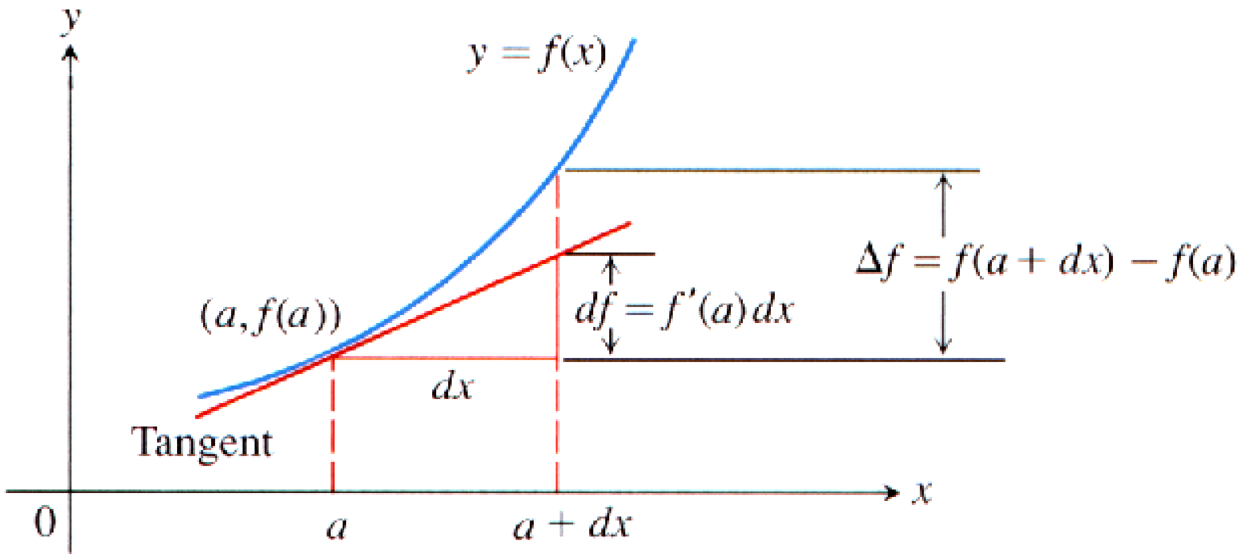
$$dy = f'(x)\,dx.$$

Example. Page 251 number 28 and 38.

Note. Differential Estimate of Change.

Let $f(x)$ be differentiable at $x = a$. The approximate change in the value of f when x changes from a to $a + dx$ is

$$df = f'(a)\,dx.$$



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Definition. We can compare actual changes in a function and the estimated change which is calculated from the use of differentials. We consider the absolute, relative, and percentage change:

	True	**Estimated**
Absolute change	$\Delta f = f(a + dx) - f(a)$	$df = f'(a)\,dx$
Relative change	$\dfrac{\Delta f}{f(a)}$	$\dfrac{df}{f(a)}$
Percentage change	$\dfrac{\Delta f}{f(a)} \times 100\%$	$\dfrac{df}{f(a)} \times 100\%$

Example. Page 252 number 56 and 58.