Chapter 4. Applications of Derivatives

4.1 Extreme Values of Functions

Definition. Let f be a function with domain D. Then $f(c)$ is the

(a) *absolute maximum value* on D if and only if $f(x) \leq f(c)$ for all x in D

(b) *absolute minimum value* on D if and only if $f(x) \geq f(c)$ for all x in D.

Theorem 1.The Extreme-Value Theorem for Continuous Functions

If f is continuous at every point of a closed and bounded interval $I = [a, b]$, then f assumes both an absolute maximum value M and an absolute minimum value m somewhere in I. That is, there are numbers x_1 and x_2 in $I = [a, b]$ with $f(x_1) = m$, $f(x_2) = M$, and $m \leq f(x) \leq M$ for every x in $I = [a, b]$.

Examples. Page 272 numbers 2 and 4.

Definition. Let c be an interior point of the domain of the function f. Then $f(c)$ is a

(a) *local maximum value* if and only if $f(x) \leq f(c)$ for all x in some open interval containing c

(b) *local minimum value* if and only if $f(x) \geq f(c)$ for all x in some open interval containing c.

Theorem 2. Local Extreme Values.

If a function f has a local maximum value or a local minimum value at an interior point c of its domain, and if f' exists at c, then $f'(c) = 0$.

Definition. A point in the domain of a function f at which $f' = 0$ or f' does not exist is a *critical point* of f.

Note. How to Find the Absolute Extrema of a Continuous Function f on a Closed Interval

To find extrema on a closed and bounded interval, we first find the critical points and then:

Step 1. Evaluate f at all critical points and endpoints.

Step 2. Take the largest and smallest of these values.

Examples. Page 273 numbers 18, and 54, page 274 numbers 58 and 76.