Chapter 4. Applications of Derivatives

4.4 Concavity and Curve Sketching

Definition. The graph of a differentiable function $y = f(x)$ is

(a) *concave up* on an open interval I if y' is increasing on I

(b) *concave down* on an open interval I if y' is decreasing on I.

Note. Second Derivative Test for Concavity.

The graph of a twice-differentiable function $y = f(x)$ is

(a) concave up on any interval where $y'' > 0$

(b) concave down on any interval where $y'' < 0$.

Note. If f is concave up at point (x_0, y_0), then a tangent line to f at (x_0, y_0) lies **below** the graph of f near (x_0, y_0). If f is concave down at point (x_0, y_0), then a tangent line to f at (x_0, y_0) lies **above** the graph of f near (x_0, y_0).

Definition. A point where the graph of a function has a tangent line and where the concavity changes is a *point of inflection.*

Example. Page 299 number 12.

Theorem 5. Second Derivative Test for Local Extrema.

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.

2. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

Note. Strategy for Graphing $y = f(x)$.

1. Identify the domain of f and any symmetries the curve may have.

2. Find y' and y''.

3. Find the critical points of f, and identify the function's behavior at each one.

4. Find where the curve is increasing and where it is decreasing.

5. Find the points of inflection, if any occur, and determine the concavity of the curve.

6. Identify any asymptotes.

7. Plot key points, such as the intercepts and the points found in Steps 3–5, and sketch the curve.

Example. Page 299 number 56, page 301 number 84.