Chapter 4. Applications of Derivatives

4.7 Newton's Method

Note. In Newton's Method, we try to approximate an x-intercept of a graph. We use tangent lines to improve our approximations as follows:

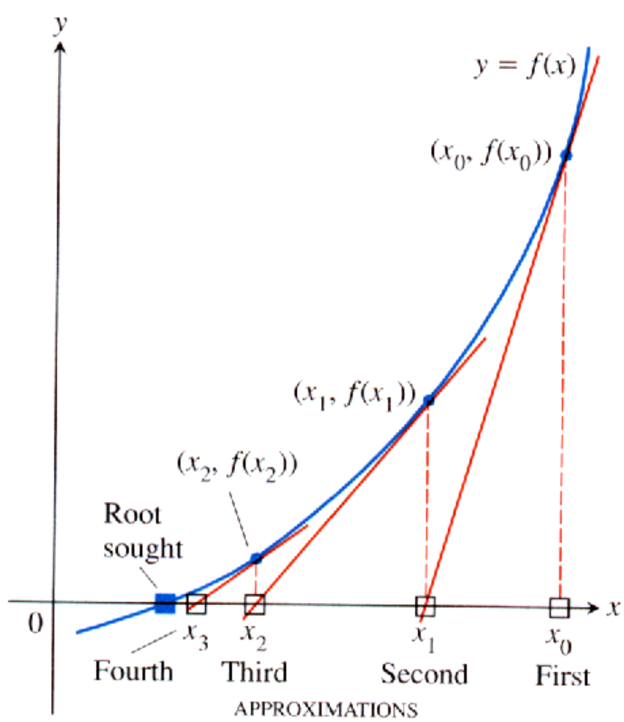

Figure 4.45, page 326

Note. Procedure for Newton's Method

1. Guess a first approximation to a solution of the equation $f(x) = 0$.

2. Use the first approximation to get a second, the second to get a third, and so on, using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Note. The equation of the tangent line to $y = f(x)$ at $(x_n, f(x_n))$ is $y - f(x_n) = f'(x_n)(x - x_n)$. Therefore the intercept x_{n+1} satisfies

$$0 - f(x_n) = f'(x_n)(x_{n+1} - x_n)$$

$$\text{or } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

(see page 326 for the algebra steps).

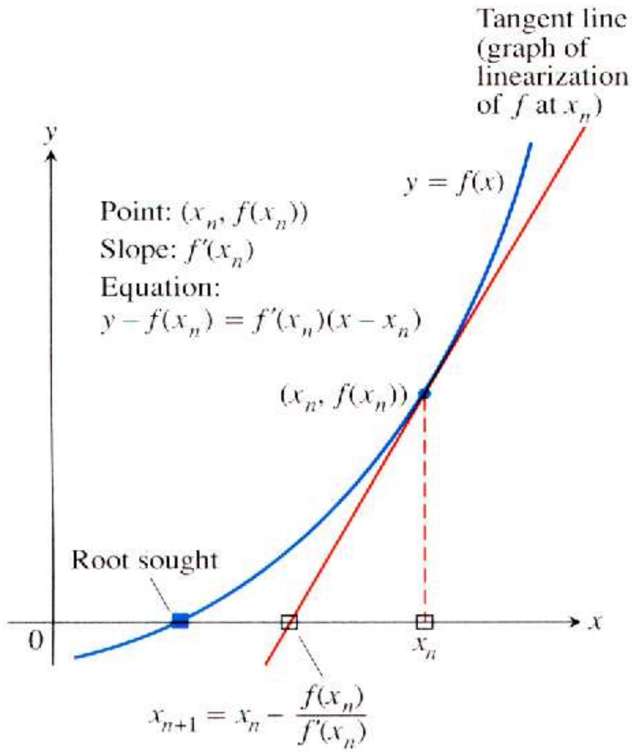

Figure 4.46, page 326

Examples. Page 329 number 2, page 330 number 18.