# Chapter 5. Integration

# 5.5 Indefinite Integrals and the Substitution Rule

**Note.** Suppose we denote a function $f(x)$ as $u$: $u = f(x)$. Then, for $n$ a rational number, $n \neq -1$, by the Power Rule and Chain Rule for differentiation, $\dfrac{d}{dx}\left[\dfrac{u^{n+1}}{n+1}\right] = \dfrac{d}{dx}\left[\dfrac{(f(x))^{n+1}}{n+1}\right] = (f(x))^n f'(x)$. As an indefinite integral, we can write this as $\int (f(x))^n f'(x)\, dx = \dfrac{(f(x))^{n+1}}{n+1} + C$. Since the differential of $u$ is $du = f'(x)\, du$, we write $\int u^n\, du = \dfrac{u^{n+1}}{n+1} + C$.

**Example.** Page 402 number 18.

**Theorem 5. The Substitution Rule.** If $u = g(x)$ is a differentiable function whose range is an interval $I$ and $f$ is continuous on $I$, then

$$\int f(g(x))g'(x)\, dx = \int f(u)\, du.$$

**Proof.** This can be established by the Chain Rule. *QED*

**Examples.** Page 403 numbers 44 and 48. Page 400 Example 9, page 403 numbers 60 and 68.