Chapter 5. Integration

5.6 Substitution and Area Between Curves

Note. We can use u-substitution in definite integrals:

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)\, du$$

where $u = g(x)$, and $du = g'(x)\, dx$.

Examples. Page 410 numbers 14a and 18.

Definition. If f and g are continuous with $f(x) \geq g(x)$ throughout $[a, b]$, then the *area* of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $[f - g]$ from a to b:
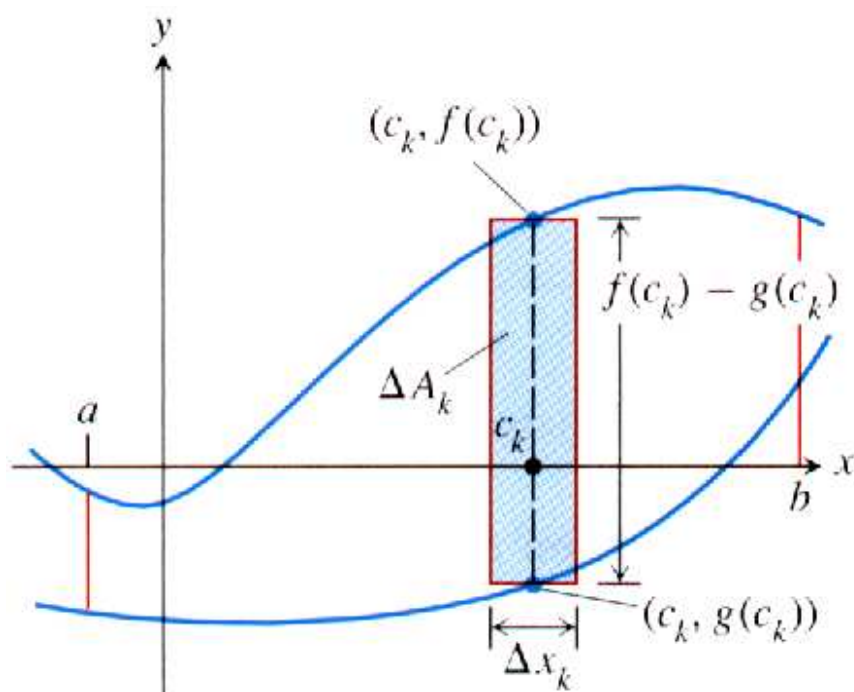
$$A = \int_a^b [f(x) - g(x)]\, dx.$$

Figure 5.29, page 407

Note. We will take a heuristic shortcut and take "*dx*" slices.

Examples. Page 412 numbers 56 and 66, page 413 number 102.