Chapter 6. Applications of Definite Integrals

6.1 Volumes by Slicing and Rotation About an Axis

Definition. The *volume* of a solid of known integrable cross-section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b,

$$V = \int_a^b A(x)\,dx.$$

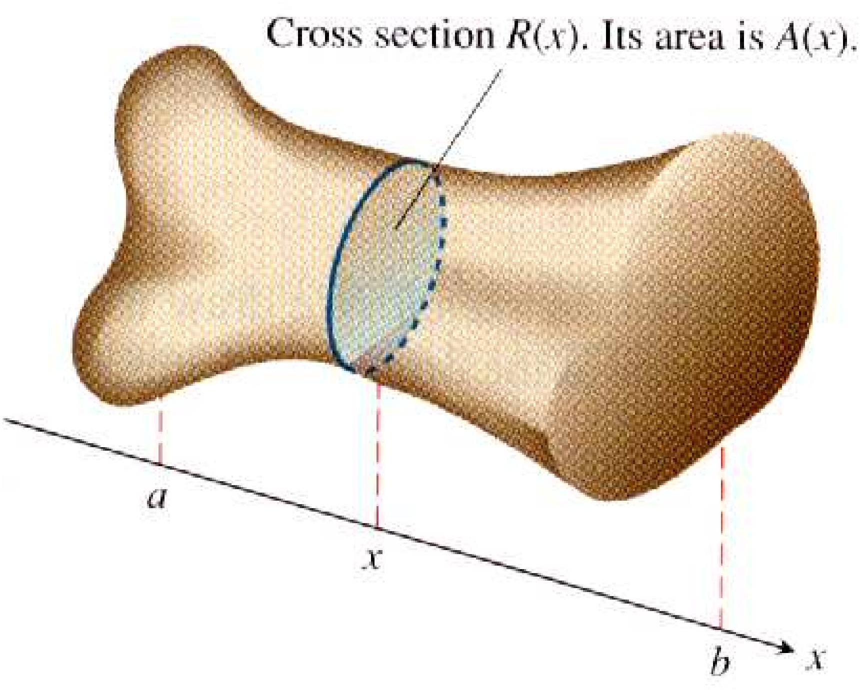


Figure 5.1, page 393 of Edition 10

Example. Page 434 number 2b.

Note. If we revolve an area about an axis, then we get cross sectional areas which are circular. We are lead to the "disk method."

Note. Disk Method for Rotation about the x-Axis

The volume of the solid generated by revolving about the x-axis the region between the x-axis and the graph of the continuous function $y = R(x)$, $a \le x \le b$, is

$$V = \int_a^b \pi[\text{radius}]^2\, dx = \int_a^b \pi[R(x)]^2\, dx.$$

We can make a similar definition for $x = R(y)$ and rotation about the y-axis.

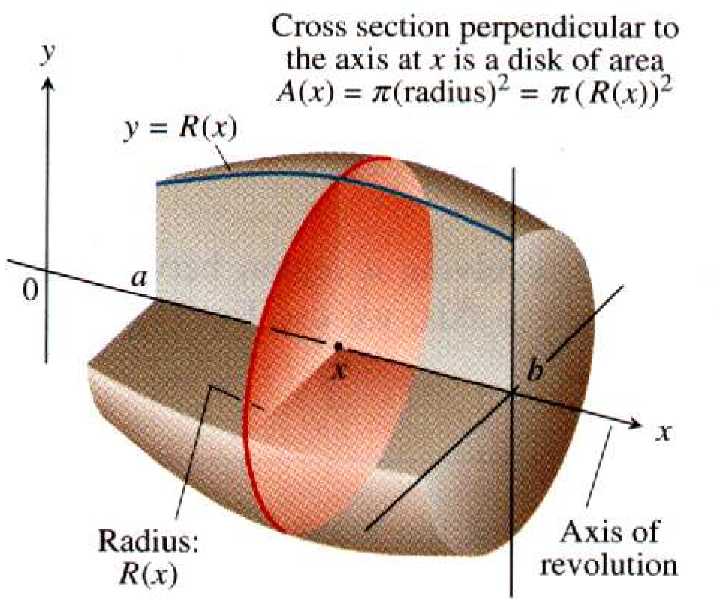


Figure 1.14, page 379 of 9th Edition

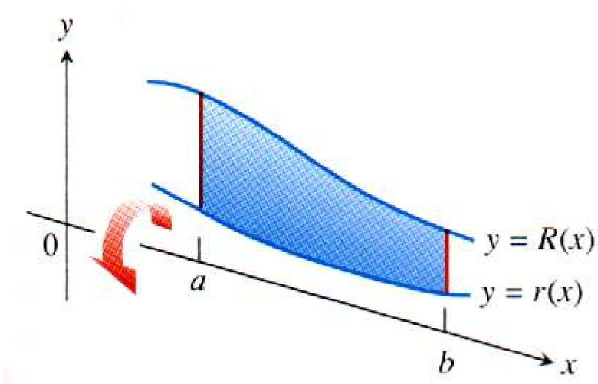
Example. Page 436 number 16.

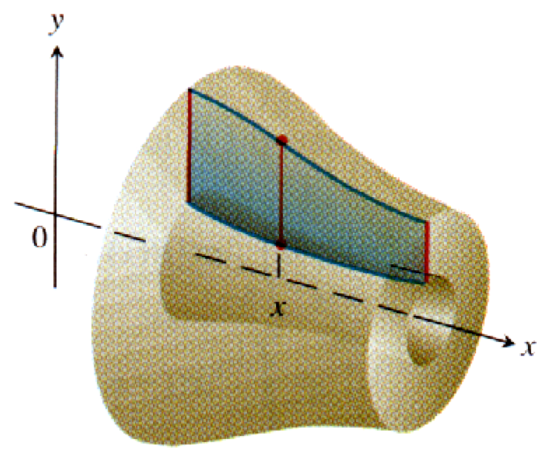
Note. If the revolution does not result in disks, but results in disks with holes in the centers, then we are lead to the "washer method."

Note. Washer Method for Rotation about the x-Axis.
The volume of the solid generated by revolving about the x-axis the region between $y = r(x)$ and $y = R(x)$ where $0 \leq r(x) \leq R(x)$ and $r(x), R(x)$ are continuous, for $a \leq x \leq b$ is

$$V = \int_a^b \pi[(\text{outer radius})^2 - (\text{inner radius})^2]\,dx = \int_a^b \pi[(R(x))^2 - (r(x))^2]\,dx.$$

We can make a similar definition for a function of y and rotation about the y-axis.





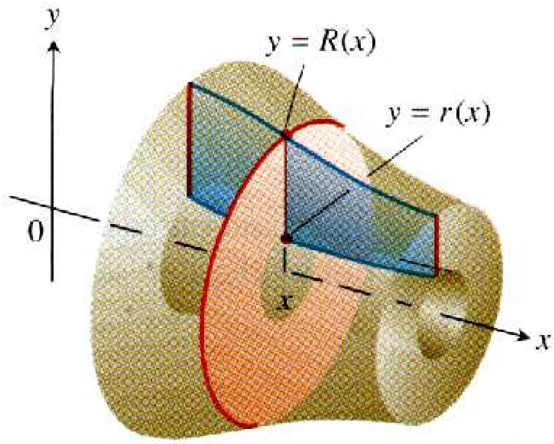


Figure 6.13, page 432

Example. Page 436 number 42, page 437 number 60.