Chapter 6. Applications of Definite Integrals

6.1 Volumes Using Cross-Sections

Definition. The *volume* of a solid of known integrable cross-section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b,

$$V = \int_a^b A(x)\,dx.$$

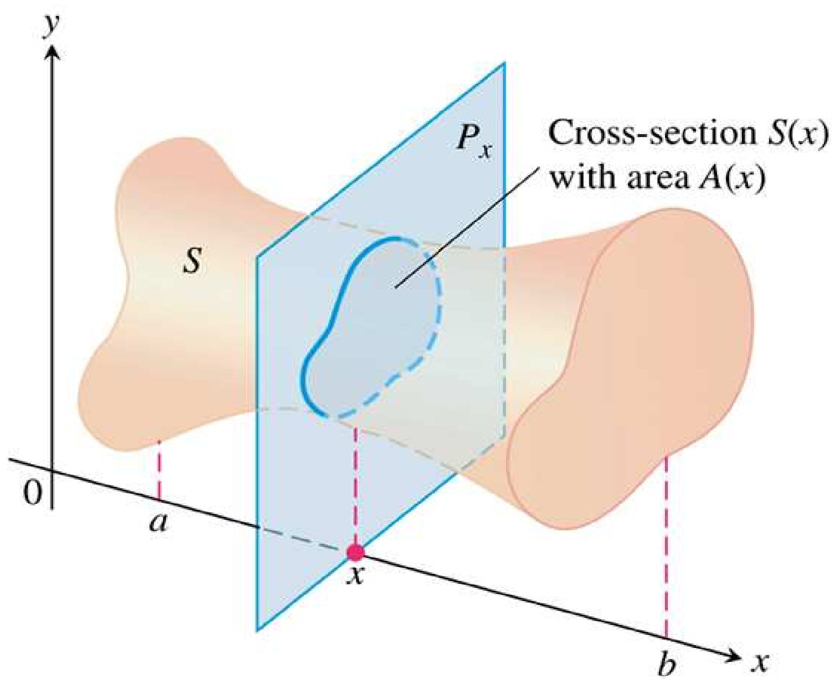

Figure 6.1, page 363

Example. Page 371 number 2.

Note. If we revolve an area about an axis, then we get cross sectional areas which are circular. We are lead to the "disk method."

Note. Disk Method for Rotation about the x-Axis

The volume of the solid generated by revolving about the x-axis the region between the x-axis and the graph of the continuous function $y = R(x)$, $a \leq x \leq b$, is

$$V = \int_a^b \pi[\text{radius}]^2\, dx = \int_a^b \pi[R(x)]^2\, dx.$$

We can make a similar definition for $x = R(y)$ and rotation about the y-axis.

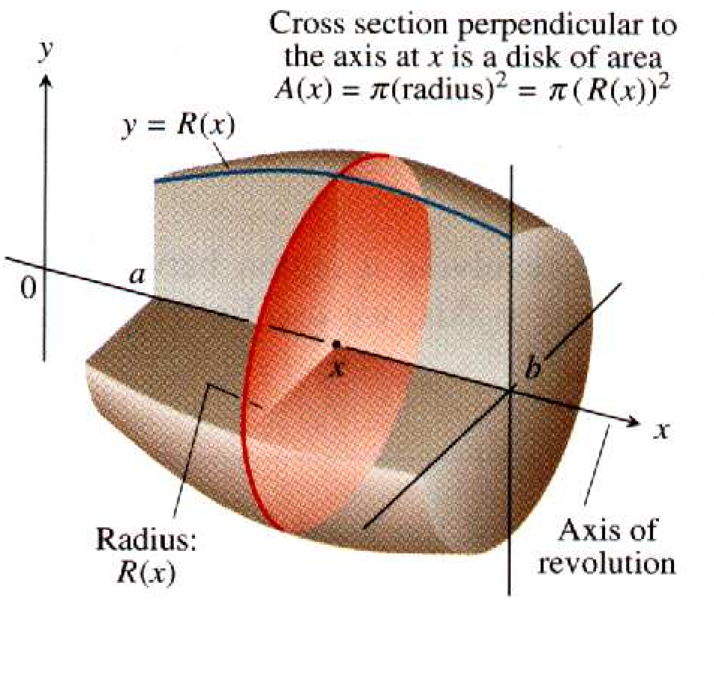

Figure 1.14, page 379 of 9th Edition

Example. Page 372 number 16.

Note. If the revolution does not result in disks, but results in disks with holes in the centers, then we are lead to the "washer method."

Note. Washer Method for Rotation about the x-Axis.
The volume of the solid generated by revolving about the x-axis the region between $y = r(x)$ and $y = R(x)$ where $0 \le r(x) \le R(x)$ and $r(x), R(x)$ are continuous, for $a \le x \le b$ is

$$V = \int_a^b \pi[(\text{outer radius})^2 - (\text{inner radius})^2]\, dx = \int_a^b \pi[(R(x))^2 - (r(x))^2]\, dx.$$

We can make a similar definition for a function of y and rotation about the y-axis.

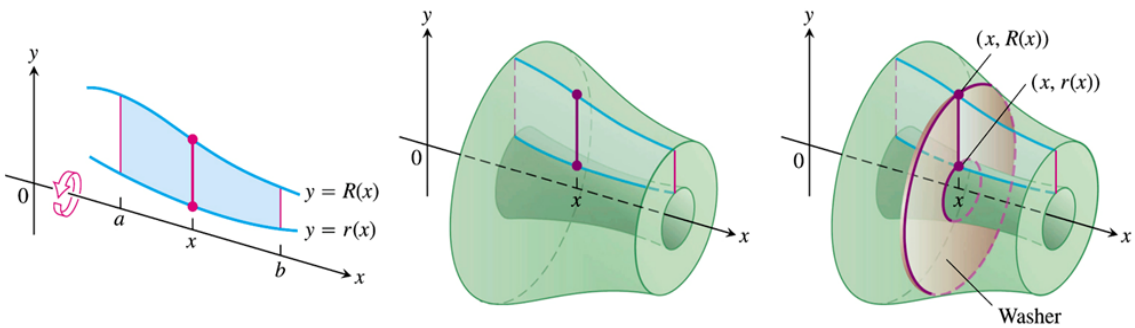

Figure 6.13, page 369

Example. Page 372 number 42, page 373 number 60.