Chapter 6. Applications of Definite Integrals

6.3 Arc Length

Definition. If a curve C is described by the parametric equations $x = f(t)$, $y = g(t)$, $\alpha \leq t \leq \beta$, where f' and g' are continuous and not simultaneously zero on $[\alpha, \beta]$ and if C is traverses exactly once as t increases from α to β, then the length of C is

$$\begin{aligned} L &= \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt. \\ &= \int_\alpha^\beta \sqrt{(f'(t))^2 + (g'(t))^2}\, dt. \end{aligned}$$

Examples. Page 452 number 2.

Definition. Function f is *smooth* if it's derivative in continuous. If f is smooth on $[a, b]$, the *length* of the curve $y = f(x)$ from a to b is the number

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

If g is smooth on $[c, d]$, the *length* of the curve $x = g(y)$ from c to d is the number

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy.$$

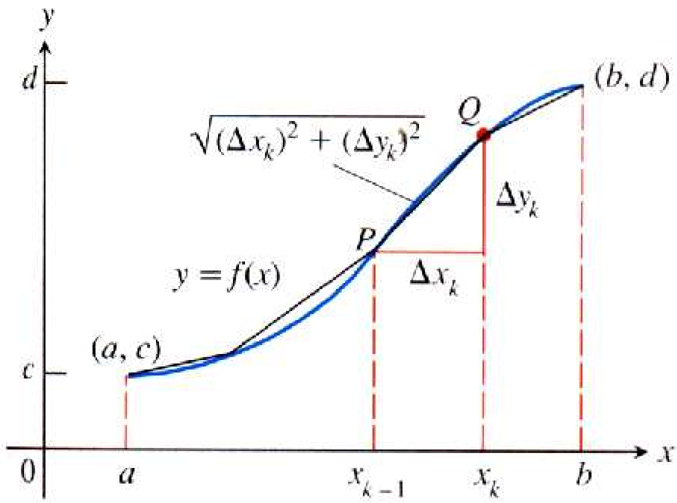

Figure 5.27, page 415 of Edition 10

Definition. In general, the *differential of arc length* is $ds = \sqrt{dx^2 + dy^2}$.

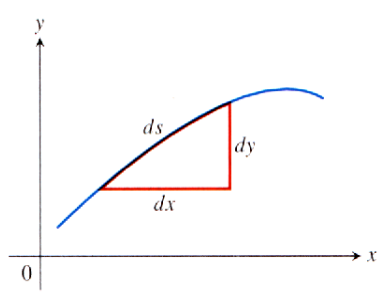

Figure 5.29, page 418 of Edition 10

Examples. Page 386 numbers 6, 10, 20.