# **Chapter 6.** Applications of Definite Integrals

## **6.4.** Areas of Surfaces of Revolution

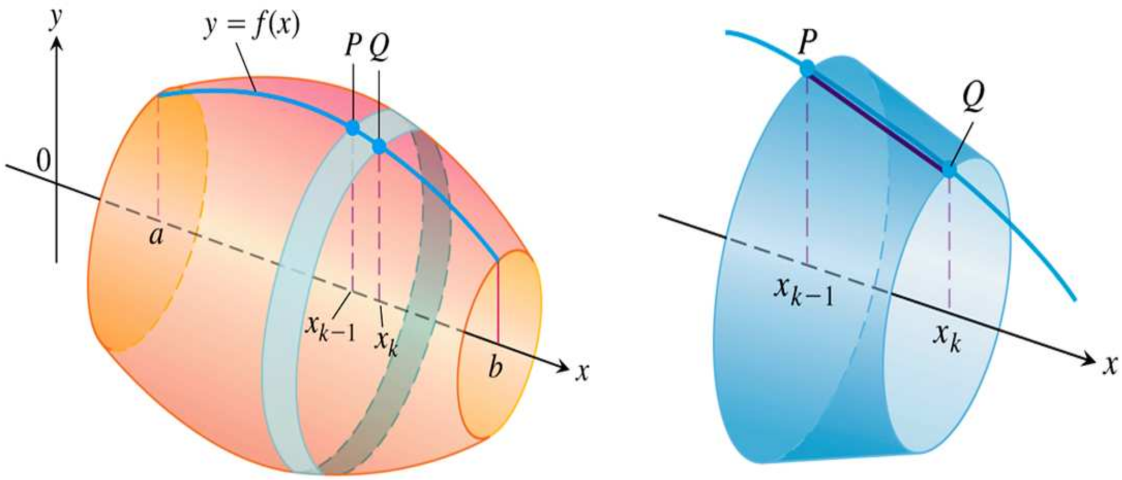
**Recall.** A differential of arclength $s$ of the function $y = f(x)$ is

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

**Example.** Set up an integral for the length of $y = x^2$ and $x \in [0, 1]$. Notice that you cannot evaluate the resulting integral. This innocent looking integral will require several of the techniques developed in Chapter 8.

**Note.** In order to find the surface area that results from revolving an arc, we partition the arc into pieces and revolve them. If we follow the "heuristic" approach (as opposed to explicitly going through the partition, the slices, the norm of the partition, and limits), then a slice of arclength produces a ring of width $ds$ and radius $x$ (if the arc was revolved about the $y$-axis) or $y$ (if the arc was revolved about the $x$-axis). Therefore the area of the ring, which is a differential of the area of the surface, is

$dS = 2\pi \times$ radius $\times\, ds$.




Figures 6.30 and 6.31, page 389

**Definition.** If the function $f(x) \geq 0$ is continuously differentiable on $[a, b]$, the *area* of the surface generated by revolving the curve $y = f(x)$ about the $x$-axis is

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx.$$

**Example.** Page 391 number 14.

**Definition.** If $x = g(y) \geq 0$ is continuously differentiable on $[c, d]$, the area of the surface generated by revolving the curve $x = g(y)$ about the $y$-axis is

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_c^d 2\pi g(y)\sqrt{1 + (g'(y))^2}\, dy.$$

**Examples.** Page 391 number 8; page 393 number 32.