Chapter 6. Applications of Definite Integrals

6.5. Work and Fluid Forces

Recall. When a body moves a distance d along a straight line as a result of being acted on by a force of constant magnitude F in the direction of motion, then the *work* done is $W = Fd$. If we measure forces in pounds and distances in feet, then work is measured in foot-pounds. If we measure forces in Newtons and distances in meters, then work is measured in newton-meters, and one newton-meter equals one *Joule*.

Definition. The *work* done by a variable force $F(x)$ directed along the x-axis from $x = a$ to $x = b$ is

$$W = \int_a^b F(x)\,dx.$$

Note. If we imagine partitioning the x-axis into a bunch of little dx slices, then the work done over one of these slices located at position x is then force $\times$ distance $= F(x)dx$. Integrating over all x values gives the definition above. (This is consistent with the usual informal interpretation of definite integrals as sums of dx [or dy] slices.)

Example. Page 399 number 8.

Recall. Hooke's Law states that the force it takes to stretch or compress a spring x length units from its natural (unstressed) length is proportional to x. That is, $F(x) = kx$ where k is the *spring constant* and carries unites of force per unit length.

Example. Page 398 number 4.

Note. We will consider several work problems which involve pumping a liquid out of a tank. We will describe the sides of the tanks by a function of y, take dy slices of the fluid, find the (1) volume of the slice, (2) weight of the slice, (3) work done on the slice, and integrate up the work done on the slices to find the total work. We must take dy slices since the only work done is force against the pull of gravity, and therefore motion along the y-axis.

Example. Page 400 number 22.

Recall. The pressure p at depth h in a fluid of weight-density w is $p = wh$ (assuming the fluid is stationary — a moving fluid will exert less pressure; that's how an airfoil works).

Definition. Suppose that a plate submerged vertically in fluid of weight-density w runs from $y = a$ to $y = b$ on the y-axis. Let $L(y)$ be the length of the horizontal strip measured from left to right along the surface of the plate at level y. Then the force exerted by the fluid against one side of the plate is

$$F = \int_a^b w(\text{strip depth})L(y)\,dy.$$

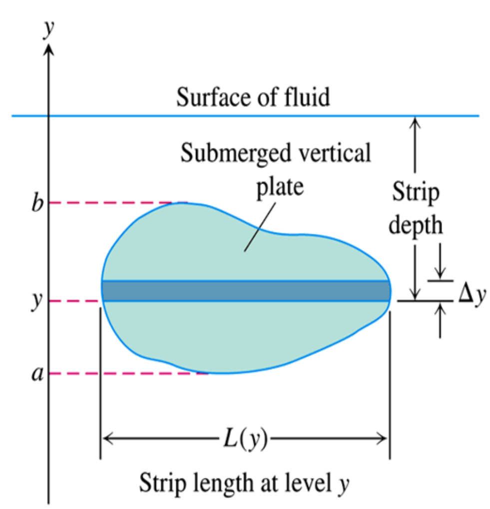

Figure 6.42, page 397.

Examples. Page 401 numbers 34 and 40.