Chapter 8. Techniques of Integration

8.3 Trigonometric Substitution

Note. In this section, we consider integrals involving expressions of the forms

1. $a^2 + x^2$

2. $a^2 - x^2$

3. $x^2 - a^2$

in "inconvenient" places (such as under square root radicals). We make the following substitutions in each case:

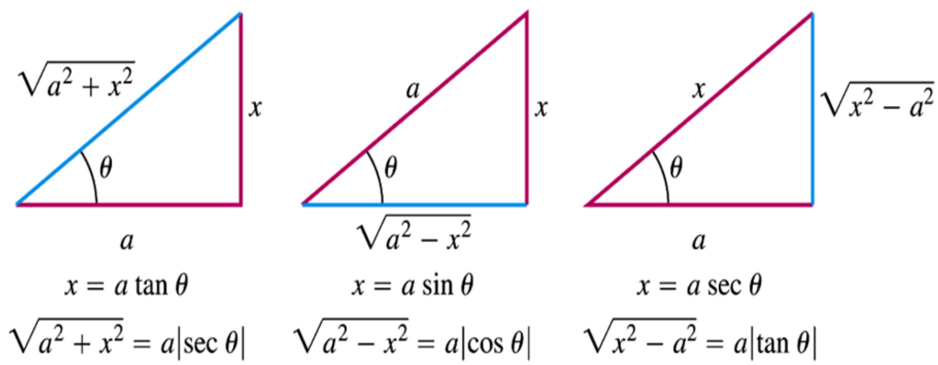
x	expression E	E becomes	dx
$a\tan\theta$	$a^2 + x^2$	$a^2\sec^2\theta$	$a\sec^2\theta\,d\theta$
$a\sin\theta$	$a^2 - x^2$	$a^2\cos^2\theta$	$a\cos\theta\,d\theta$
$a\sec\theta$	$x^2 - a^2$	$a^2\tan^2\theta$	$a\sec\theta\tan\theta\,d\theta$

Note. After we convert from x values (say) to trig functions of θ (say), we will eventually need to reverse the process and convert back to x's. We will do so using a "reference triangle." We will find:

$x = a\tan\theta$ requires $\theta = \tan^{-1}\left(\dfrac{x}{a}\right)$ with $\theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$,

$x = a\sin\theta$ requires $\theta = \sin^{-1}\left(\dfrac{x}{a}\right)$ with $\theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$,

$x = a\sec\theta$ requires $\theta = \sec^{-1}\left(\dfrac{x}{a}\right)$ with $\begin{cases} \theta \in \left[0, \frac{\pi}{2}\right), & \frac{x}{a} \geq 1 \\ \theta \in \left(\frac{\pi}{2}, \pi\right], & \frac{x}{a} \leq -1 \end{cases}$



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Examples. Page 470 numbers 8 and 36, page 471 number 51.

Example. Find the arclength of the graph of $y = x^2$ for $x \in [0, 1]$.