Chapter 9. First-Order Differential Equations

9.4 Graphical Solutions of Autonomous Equations

Definition. An equation of the form $\dfrac{dy}{dx} = g(y)$ is an autonomous ordinary differential equation.

Definition. If $\dfrac{dy}{dx} = g(y)$ is an autonomous differential equation, then the values of y for which $\dfrac{dy}{dx} = 0$ are called *equilibrium values* or *rest points*.

Example. Page 540 number 2a.

Note. We make use of a *phase line* for these types of differential equations. This is a plot on the y-axis that shows the equation's equilibrium values along with the intervals where dy/dx and d^2y/dx^2 are positive and negative. Then we know where the solutions are increasing and decreasing and the concavity of the solution curves.

Example. Page 540 number 2b.

Note. An equilibrium value is *stable* if we perturb the system slightly and it returns to the equilibrium value. For example, a ball at the bottom of a well is in a stable equilibrium state — perturb the ball slightly and it returns to where it was. An equilibrium value is *unstable* if a small perturbation of the system may yield solutions that do not return to the equilibrium value. For example, a ball at the top of a hill is in an unstable equilibrium — perturb the ball a little and it rolls away.

Example. Page 540 number 2a (continued), 2c.

Example. Page 539: Consider the logistic equation

$$\frac{dP}{dt} = r(M - P)P.$$

Present graphical solutions. The text also has examples which graphically explore Newton's Law of Cooling and problems involving terminal velocity of a falling object under air resistance.