Calculus 2 Test 1 — Summer 2011

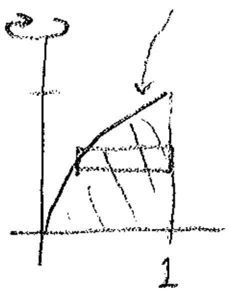
NAME KEY STUDENT NUMBER ____________________

SHOW ALL WORK!!! Do not rely on the calculators. Include all necessary symbols (such as equal signs). When applicable, **draw the region** mentioned in the problem **and the resulting solid or surface**. The more details you show, the easier it will be to give you partial credit (if needed). Notice that some problems just ask you to *set up* the integrals for the solutions. Put your final answer in the box provided, or put a box around your final answer. Each numbered problem is worth 12 points.

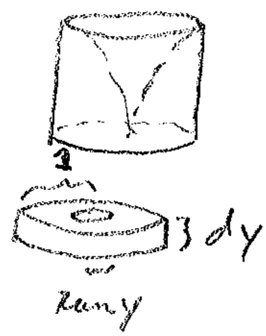
p372 #38

BELOW CURVE!

1. The curve $x = \tan y$ for $y \in [0, \pi/4]$ is revolved about the y-axis. **Find** the resulting volume.



Revolve about y-axis



1 dy-slice

$(\text{volume}) = \pi(1)^2 dy - \pi(\tan y)^2 dy$

$= \pi(1 - \tan^2 y)\,dy$

so

$$V = \int_0^{\pi/4} \pi(1 - \tan^2 y)\,dy = \int_0^{\pi/4} \pi\left(1 - (\sec^2 y - 1)\right)dy = \int_0^{\pi/4} \pi(2 - \sec^2 y)\,dy$$

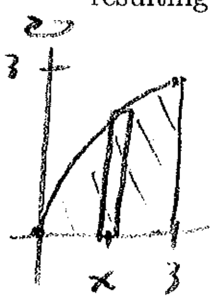
$$= \pi(2y - \tan y)\Big|_0^{\pi/4} = \pi\left(2\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right)\right) - 0$$

$$\boxed{= \pi\left(\frac{\pi}{2} - 1\right)}$$

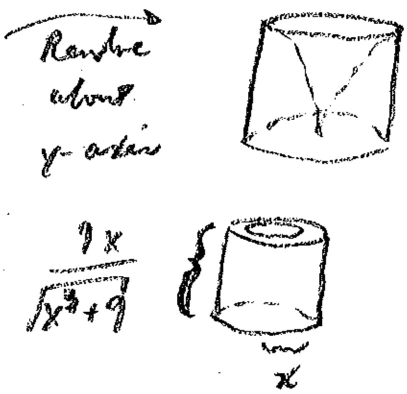
p379 #6

BELOW CURVE!

2. The curve $y = \dfrac{9x}{\sqrt{x^3+9}}$ for $x \in [0, 3]$ is revolved about the y-axis. **Set up an integral** for the resulting volume.



Revolve about y-axis



1 slice:

$\text{volume} = 2\pi x \, \dfrac{9x}{\sqrt{x^3+9}}\,dx$

1 solid,

$$\boxed{V = \int_0^3 2\pi x \, \frac{9x}{\sqrt{x^3+9}}\,dx}$$

p 386 #10

3. **Set up an integral** for the length of the curve $y = \int_{-2}^{x} \sqrt{3t^4 - 1}\,dt$ for $x \in [-2, -1]$.

Well, $\dfrac{dy}{dx} = \dfrac{d}{dx}\left[\int_{-2}^{x} \sqrt{3t^4 - 1}\,dt\right] = \sqrt{3x^4 - 1}$ by F.T.C. I

So

$$L = \int_{-2}^{-1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_{-2}^{-1} \sqrt{1 + \left(\sqrt{3x^4 - 1}\right)^2}\,dx$$

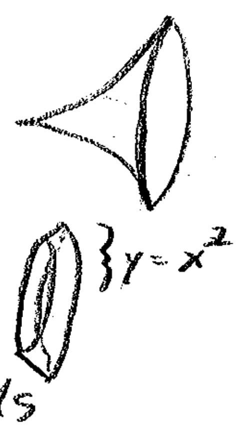
$$= \int_{-2}^{-1} \sqrt{1 + (3x^4 - 1)}\,dx = \int_{-2}^{-1} \sqrt{3x^4}\,dx$$

$$\boxed{= \int_{-2}^{-1} \sqrt{3}\, x^2\,dx}$$

p 399 #2

4. The curve $y = x^2$ for $x \in [0, 2]$ is revolved about the x-axis. **Set up an integral** for the resulting surface area.

Revolve about x-axis



$$(\text{Area of slice}) = 2\pi (x^2)\, ds$$

$$= 2\pi x^2 \sqrt{1 + (2x)^2}\,dx$$

$$= 2\pi\, x^2 \sqrt{1 + 4x^2}\,dx$$

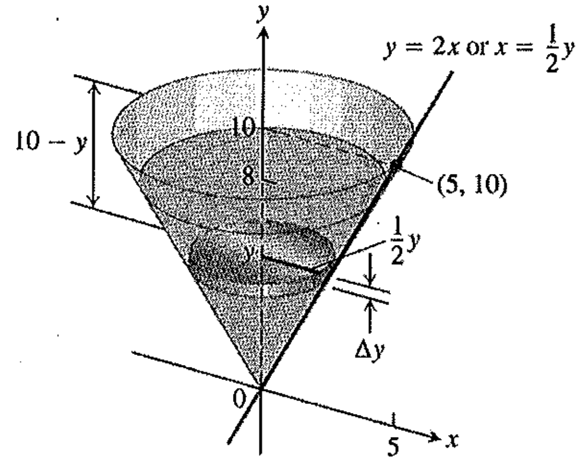
$$\boxed{\int_0^2 2\pi x^2 \sqrt{1 + 4x^2}\,dx}$$

So total surface area is

$$A = \int_0^2 2\pi x^2 \sqrt{1 + 4x^2}\,dx$$

p396 eg 5

5. The conical tank below is filled to within 2 ft of the top with olive oil weighing 57 lb/ft^3. **Set up an integral** for the work it takes to pump the oil to the rim of the tank.



$x = \frac{1}{2}y$

dy

for dy-slice:

$$(\text{volume}) = \pi\left(\tfrac{1}{2}y\right)^2 dy = \frac{\pi y^2}{4}\,dy \quad (\text{ft}^3)$$

$$(\text{force}) = 57\,\frac{\pi y^2}{4}\,dy \quad (\text{lb})$$

$$(\text{distance}) = 10 - y \quad (\text{ft})$$

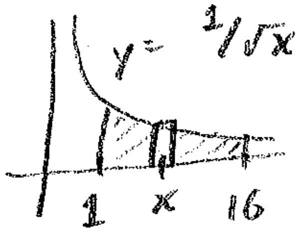
$$\text{work} = 57\,\frac{\pi y^2}{4}(10-y)\,dy \quad (\text{ft lb})$$

So, total work is

$$W = \int_0^8 57\,\frac{\pi y^2}{4}(10-y)\,dy$$

p 412 #13

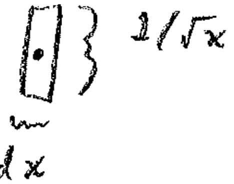
6. Consider the region between the curve $y = 1/\sqrt{x}$ and the x-axis for $x \in [1, 16]$. For this region, **set up integrals** for the moment about the x-axis (M_x), the moment about the y-axis (M_y), and the mass M. Express the center of mass of the region in terms of M_x, M_y, and M.



For dx-slice:

$$(\text{mass}) = (\text{density})(\text{area}) = \delta \frac{1}{\sqrt{x}}\,dx$$

$$(\tilde{x}, \tilde{y}) = \left(x, \frac{1}{2\sqrt{x}}\right)$$



$$\begin{pmatrix}\text{moment}\\ \text{about } x\text{-axis}\end{pmatrix} = \tilde{y}\,(\text{mass}) = \frac{1}{2\sqrt{x}}\,\delta\,\frac{1}{\sqrt{x}}\,dx = \frac{\delta}{2x}\,dx$$

$$\begin{pmatrix}\text{moment}\\ \text{about } y\text{-axis}\end{pmatrix} = \tilde{x}\,(\text{mass}) = x\,\delta\,\frac{1}{\sqrt{x}}\,dx = \delta\sqrt{x}\,dx$$

So: $$M = \int_1^{16} \delta \frac{1}{\sqrt{x}}\,dx$$

$$M_x = \int_1^{16} \frac{\delta}{2x}\,dx$$

$$M_y = \boxed{\int_1^{16} \delta\sqrt{x}\,dx, \quad \bar{x} = \frac{M_y}{M}, \quad \bar{y} = \frac{M_x}{M}}$$

7. (a) **Evaluate the integral** $\displaystyle\int \frac{2y\,dy}{y^2 - 25}$.

p 425 #3

let $u = y^2 - 25$

$du = 2y\,dy$

$$= \int \frac{1}{u}\,du = \ln|u| + C$$

$$= \ln|y^2 - 25| + C$$

$$\boxed{\ln|y^2 - 25| + C}$$

p448 #3d

(b) Does the function $g(x) = (x+3)^2$ grow faster than, grow slower than, or grow at the same rate as $f(x) = x^2$? Justify your answer.

Well, $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{x^2}{(x+3)^2} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{2x}{2(x+3)}$

$\overset{\infty/\infty}{=} \lim_{x\to\infty} \left(\frac{2}{2}\right) = 1 = L.$

$\boxed{\text{f and g grow at the same rate}}$

8. Solve the separable differential equation: $\sqrt{x}\frac{dy}{dx} = e^{y+\sqrt{x}}$, $x > 0$.

p434 #15

$\sqrt{x}\,\frac{dy}{dx} = e^y e^{\sqrt{x}}$

$e^{-y}\,dy = \frac{1}{\sqrt{x}} e^{\sqrt{x}}\,dx$

$\int e^{-y}\,dy = \int \frac{1}{\sqrt{x}} e^{\sqrt{x}}\,dx$

$-e^{-y} = 2e^{\sqrt{x}} + C$

$e^{-y} = -2e^{\sqrt{x}} - k$

$\ln(e^{-y}) = \ln(-2e^{\sqrt{x}} - k)$

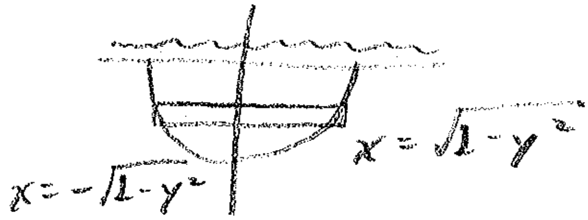
$-y = \ln(-2e^{\sqrt{x}} - k)$

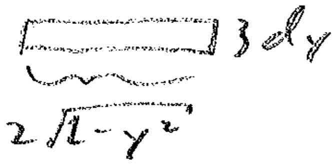
$\boxed{y = -\ln(-2e^{\sqrt{x}} - k)}$

Bonus. A semicircular plate 2 ft in diameter sticks straight down into freshwater with the diameter along the surface. **Find the force** exerted by the water on one side of the plate. The weight-density of water is 62.4 lb/ft^3.

p399 #40

$\boxed{\frac{124.8}{3} \text{ lb} = 41.6 \text{ lb}}$





For dy slice:

$$(\text{area}) = 2\sqrt{1-y^2}\,dy \quad (\text{ft}^2)$$

$$(\text{depth}) = 0 - y = -y \quad (\text{ft})$$

$$(\text{pressure}) = \begin{pmatrix}\text{weight}\\ \text{density}\end{pmatrix}(\text{depth}) = 62.4(-y) \quad (\text{lb}/\text{ft}^2)$$

$$(\text{force}) = (\text{pressure})(\text{area}) = -62.4y\; 2\sqrt{1-y^2}\,dy = -124.8\,y\sqrt{1-y^2}\,dy \quad (\text{lb})$$

So

$$F = \int_{-1}^{0} -124.8\, y\sqrt{1-y^2}\,dy$$

let $u = 1 - y^2$

$du = -2y\,dy$

$\frac{du}{-2} = y\,dy$

$$= \int_{y=-1}^{y=0} -124.8\sqrt{u}\,\frac{du}{-2} = 62.4\int_{y=-1}^{y=0} u^{1/2}\,du$$

$$= 62.4\left(\tfrac{2}{3}u^{3/2}\right)\Big|_{y=-1}^{y=0} = \frac{124.8}{3}(1-y^2)^{3/2}\Big|_{-1}^{0}$$

$$= \frac{124.8}{3} - 0 = \boxed{\frac{124.8}{3}\ \text{lb}}$$