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A Useful Notation for Rules of Differentiation

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The freshman calculus instructor is often faced with the task of deciphering the freshman calculus student's versions of the product rule, quotient rule and chain rule. Even when correctly computed, it can be difficult to follow the student's steps, especially when the differentiated function involves several trigonometric functions. I propose a notational convention to deal with this problem.

Several popular calculus books (e.g. Swokowski, Larson et al., etc.) represent the differential operator as $D[\cdot]$ or $d/dx[\cdot]$. With this as motivation, I have used the following notation quite successfully in the classroom. Whenever a function is differentiated using the product rule, quotient rule, or chain rule, put the differentiated parts in square brackets. This means that the product rule would be written as

$$D[(f)(g)] = [f'](g) + (f)[g'].$$

In fact, if the instructor introduces the convention of "always differentiate f (the first function in the product) first," then the product rule can be presented pictorially as

$$D[(\ )(\ )] = [\ ](\ ) + (\ )[\ ].$$

This reduces exercises involving the product rule to nothing more than fill-in-the-blank problems.

Similarly, the quotient rule can be represented as

$$D\left[\frac{(\ )}{(\ )}\right] = \frac{[\ ](\ ) - (\ )[\ ]}{(\ )^2}.$$

The habit of "differentiating f first" developed in the product rule, must be carried over to the quotient rule.

The chain rule is a bit harder to "draw"; however the special case known as the power rule can be illustrated in this manner:

$$D\left[(\ )^n\right] = n(\ )^{n-1}[\ ].$$

My experience has shown that, not only is the students' work easier to follow, but the material is easier to present clearly. Students have reacted quite positively to this notation. In fact, when encouraged to adopt the notation, but not required to, I have found that practically all students choose to use the notation. This has been the case even when the notation was introduced some time after the rules of differentiation, i.e., Calculus 2 and Calculus 3 students quickly "pick up the habit."

An example illustrates the utility of this method. Suppose

$$f(x) = \frac{\sec x \tan x}{(x^2+1)^6}.$$

With the notation,

$$f'(x) = \frac{[[\sec x \tan x](\tan x) + (\sec x)[\sec^2 x]](x^2+1)^6 - (\sec x \tan x)\left[6(x^2+1)^5[2x]\right]}{(x^2+1)^{12}}.$$

Although a formidable derivative, it can be viewed as nothing more than a large fill-in-the-blank problem.

With the square brackets, it is much easier to follow the students' steps and, if applicable, to give partial credit. In fact, give the above problem on your next calculus test. I think, in the absence of the square brackets, you will find it quite a task simply to determine if the students' responses *are* partially correct!