Calculus 2, Spring 2001
TEST 4

NAME ________________________________ STUDENT NUMBER ________________________

Show all work! Include all necessary symbols (such as equal signs). The more details you show, the easier it will be to give you partial credit (if needed). **Be sure to justify all claims!** Each problem is worth 12 points.

1. Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{8\tan^{-1} n}{1+n^2}$ converges or diverges.

2. Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}$ converges or diverges.

3. Determine if the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$ converges or diverges.

4. Determine if the series $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$ converges or diverges.

5. Determine if the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$ converges absolutely, converges conditionally, or diverges.

6. Estimate the magnitude of the error involved in using the sum of the first four terms to approximate the sum of the entire series: $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.01)^n}{n}$. We will see that the actual sum is $\ln(1.01)$.

7. Describe the possible behavior of a power series $\sum_{n=1}^{\infty} c_n(x-a)^n$. That is, on what type of set might the series converge absolutely, converge conditionally and diverge.

8. Find the radius of convergence, determine where the series converges absolutely, converges conditionally, or diverges: $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n^2+3}}$.

Bonus: Show by example that $\sum(a_n b_n)$ may diverge even though $\sum a_n$ and $\sum b_n$ converge. (HINT: You might be able to use alternating series.)

Bonus: Make up a power series whose interval of convergence is $(-2, 0)$.