Bob Gardner's Quick Guide to the Use of the TI-89 Calculator in Calculus 2

INTRODUCTION

The TI-89 graphing calculator is **very** versatile! This "quick guide" discusses a few commands that will be useful in Calculus 2 applications of the calculator. Since the calculator is so versatile, you may need to refer to the *TI-89 Guidebook* for details on specific settings of your calculator (such as the `AUTO/EXACT/APPROX` modes). The page numbers mentioned below all refer to the *Guidebook*. See "Bob Gardner's Quick Guide to the Use of the TI-89 Calculator in Calculus 1" for information on TI-89 setup, evaluating limits, calculating derivatives, graphing, evaluating antiderivatives, and evaluating definite integrals.

INTEGRALS AND IMPROPER INTEGRALS

Improper integrals can be evaluated in the same way as definite integrals, only with the use of the ∞ symbol. The syntax for $\int_a^\infty f(x)\,dx$ is `∫( f(x), x, a, ∞)`. To access the integral command, press [F3] [2].

Example 1. To see that $\int_1^\infty \frac{a}{x^2}\,dx = 1$, perform these keystrokes:

[F3] [2] [X] [∧] [(−)] [2] [,] [X] [,] [1] [,] [◇] [∞] [)] [ENTER].

When dealing with rational functions, we can use the `expand(` command to convert the rational function into its partial fraction decomposition. This command is accessed by pressing [F2] [3]. See page 422.

Example 2. Show that the partial fraction decomposition of $\frac{x^4+81}{x(x^2+9)^2}$ is $\frac{1}{x} - \frac{18x}{(x^2+9^2)}$ by using the `expand(` command. Notice that is we had to calculate these partial fractions by hand, then it would require us to solve five equations in five unknowns.

DIFFERENTIAL EQUATIONS

You can graph slope fields of differential equations by setting the calculator to graph mode "`DIFF EQUATIONS`." To do so, press [MODE] [▷] [6] [ENTER]. Next, you must enter the differential equation with the [Y=] editor. Enter [◇] [Y=]. This will display a list with an initial value for the independent variable (`t0=0`), a differential equation involving the first dependent variable (`y1′=`), an initial condition for the first dependent variable (`yi′=`), and DEs and initial conditions for a second and third dependent variable. (You may need to first clear out the list with the keystrokes [F1] [8] [ENTER].) You can adjust the `t0` value if you like, and input the differential equations and initial conditions which you need.

Example 3. Put your calculator in the graph mode `DIFF EQUATION` and enter the differential equation $y' = .001y(100 - y)$ by setting `y1′` equal to `.001 y1 (100 - y1)`.

You may need to adjust the `GRAPH FORMATS`. Enter [◇] [|]. You probably want `Axes = ON`, `Labels = ON`, `Solution Method = RK` (Runga-Kutta — it's a numerical method for solving differential equations; the other option is `EULER` which is discussed in *Thomas' Calculus* in section 6.6), and `Fields = SLPFLD`. You may also need to adjust the Window Editor. These is done by entering [◇] [WINDOW]. Finally, the slope field of the DE is plotted by pressing [◇] [GRAPH]. See page 164.

Example 4. Set `GRAPH FORMATS` as described above. Adjust the Window Editor so that `tmax = 10`, `tstep = .1`, `xmin=0`, `xmax = 150`, `ymin = 0`, `ymax = 150`, `yscl = 10`, `diftol = .001`, and `fldres = 20`. Graph the slope field of the DE entered in Example 3. You should get a slope field similar to Figure 6.23 on page 515 of *Thomas' Calculus*.

You can also plot a specific solution to DE by entering an initial condition. The initial condition will be displayed as a circle and the graph of the solution will emanate from the circle.

Example 5. Go back to the `Y=` editor and enter an initial condition of `yi0 = 10`. Graph the solution. You should see a logistic curve generated on top of the slope field in Example 4.

We can also solve differential equations using the `deSolve(` command which is accessed by entering [F3] and cursoring down [▽] 12 times. This command solves a first or second order DE or IVP. The notation for entering the DE is to use the prime symbol `′` to represent derivatives. The syntax is `deSolve(` *DE*`, X, Y)` (assuming the independent variable is x and the dependent variable is y). See page 412.

Example 6. Solve the DE $y' = y$ by entering

[F3] [▽] (12 times) [ENTER] [y] [2nd] [′] [=] [y] [,] [x] [,] [y] [)].

The solution will be displayed as `y = @1·e^x`. The "@1" represents an arbitrary constant (as the calculator generates constants, it will number them consecutively as @k, until you clear home (`ClrHome`).

Example 7. Use `deSolve(` to show that the solution of $y'' = -y$ is $y = c_1 \cos x + c_2 \sin x$ where c_1 and c_2 are arbitrary constants (use two primes ′ for the second derivative).

You can solve initial value problems with the syntax `deSolve(` *DE* `and` *IC*`, X, Y)` where *IC* represents an initial condition (or two initial conditions entered as "IC_1 `and` IC_2" if the DE is second order).

Example 8. Solve the initial value problem $y'' = -y$, $y(0) = 1$, $y'(0) = 2$ by entering `deSolve( y″=-y and y(0)=1 and y′(0)=2, x, y)`. The answer is $y = \cos x + 2 \sin x$. Notice that you have to use the [alpha] key and the space key (accessed with [alpha] [(−)]).

HYPERBOLIC TRIG FUNCTIONS

The hyperbolic trig functions and their inverses can be accessed through the [MATH] menu. Enter [2nd] [5] to get to the math menu, then cursor down [▽] 11 times and hit [ENTER]. A menu appears with `sinh`, `cosh`, `tanh`, and their inverses.

SEQUENCES AND SERIES

You can explore the limit of a sequence by considering the function which generates it (see Theorem 4 of section 8.1 in *Thomas' Calculus*). This requires us to explore $\lim_{x \to \infty} f(x)$. This is done with the syntax `limit( f(x), x, ∞)`. To access the integral command, press [F3] [3].

Example 9. Show that the sequence defined by $a_n = \dfrac{2n - \ln n}{n}$ has limit 2 by exploring $\lim_{x \to \infty} \dfrac{2x - \ln x}{x}$.

We can use the `sum(` command to take finite sums or infinite sums (i.e. series). This command is accessed by pressing [F3] [4]. The syntax is `sum(` a_n`, n, 1, ∞)`. See page 483.

Example 10. We can show that $\sum_{t=1}^{\infty} \dfrac{1}{t^2} = \dfrac{\pi^2}{6}$ with the keystrokes

[F3] [4] [1] [/] [T] [∧] [2] [,] [T] [,] [1] [,] [◇] [∞] [)] [ENTER].

The TI-89 does not generate power series, but can be used to generate Taylor *polynomials* with the `taylor(` command. This command is accessed with [F3] [9]. To generate the Taylor polynomial of degree n for function $f(x)$ centered at a, the syntax is `taylor( f(x), x, n, a)`. See page 487.

Example 11. Find the Taylor polynomial of degree 8 about 0 for $f(x) = e^{-x^2}$ (you may omit the "a" term — the default is $a = 0$). The answer is $\dfrac{x^8}{24} - \dfrac{x^6}{6} + \dfrac{x^4}{2} - x^2 + 1$.