Introduction to Algebra, MATH 5127

Homework 1, Sections 0 and I.1

Due Friday September 5, 2014 at 2:30

0.A. Show that $[0,1]$ and $[0,a]$ have the same cardinality by giving a formula for a one-to-one function f mapping $[0,1]$ onto $[0,a]$. Confirm that your function is one-to-one and onto using the definitions of "one-to-one" and "onto."

0.17. Let A be a finite set with $|A| = s$. Consider Exercise 0.16 and make a conjecture about the value of the cardinality of the power set $\mathcal{P}(A)$. Prove your conjecture using Mathematical Induction.

0.18. For any set A, finite or infinite, let B^A be the set of all functions mapping A into the set $B = \{0,1\}$. Show that the cardinality of B^A is the same as the cardinality of the power set $\mathcal{P}(A)$. HINT: Each element of B^A (which assigns to each element of A, either the value 0 or 1) determines a subset A in a "natural" way (hint, hint: 1 in, 0 out).

0.34. Determine whether "$n\,\mathcal{R}\,m$ in $\mathbb{Z}^+$ if n and m have the same final digit in the usual base ten notation" is an equivalence relation. That is, check if $\mathcal{R}$ is reflexive, symmetric, and transitive. If so, describe the partition arising from the equivalence relation.

I.1.19. Find all solutions in $\mathbb{C}$ of $z^3 = -27i$. Use the polar form of a complex number, similar to the computation of roots of unity on page 18. Evaluate your solutions in terms of real and imaginary parts and evaluate all trigonometric functions.

I.1.33. Find all solutions x of the equation $x +_{12} x = 2$ in $\mathbb{Z}_{12}$ (that is, the equation $x + x \equiv 2 \pmod{12}$). Insure that you have found all solutions by checking all elements of $\mathbb{Z}_{12}$ (and showing your work).