

# Introduction to Algebra, MATH 4127

## Homework 1, Sections 0 and I.1

Due Friday September 5, 2014 at 2:30

- 0.A.** Show that  $[0, 1]$  and  $[0, a]$  have the same cardinality by giving a formula for a one-to-one function  $f$  mapping  $[0, 1]$  onto  $[0, a]$ . Confirm that your function is one-to-one and onto using the definitions of “one-to-one” and “onto.”
- 0.17.** Let  $A$  be a finite set with  $|A| = s$ . Consider Exercise 0.16 and make a conjecture about the value of the cardinality of the power set  $\mathcal{P}(A)$ . Prove your conjecture using Mathematical Induction.
- 0.34.** Determine whether “ $n \mathcal{R} m$  in  $\mathbb{Z}^+$  if  $n$  and  $m$  have the same final digit in the usual base ten notation” is an equivalence relation. That is, check if  $\mathcal{R}$  is reflexive, symmetric, and transitive. If so, describe the partition arising from the equivalence relation.
- I.1.19.** Find all solutions in  $\mathbb{C}$  of  $z^3 = -27i$ . Use the polar form of a complex number, similar to the computation of roots of unity on page 18. Evaluate your solutions in terms of real and imaginary parts and evaluate all trigonometric functions.
- I.1.33.** Find all solutions  $x$  of the equation  $x +_{12} x = 2$  in  $\mathbb{Z}_{12}$  (that is, the equation  $x + x \equiv 2 \pmod{12}$ ). Insure that you have found all solutions by checking all elements of  $\mathbb{Z}_{12}$  (and showing your work).