Introduction to Algebra, MATH 4127/5127

Homework 11, Sections IV.22 and IV.23

Due Tuesday December 9, 2014 at 3:00

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

IV.22.15. Find all zeros of $f(x)g(x)$ in $\mathbb{Z}_7$ where $f(x) = x^3 + 2x^2 + 5$ and $g(x) = 3x^2 + 2x$. Factor polynomial $f(x)g(x)$ as much as possible using the Factor Theorem (Corollary 23.3). Remember that all arithmetic is done in $\mathbb{Z}_7$ and so all coefficients you compute must be in $\{0, 1, 2, 3, 4, 5, 6\}$.

IV.22.24. Prove that if D is an integral domain, then $D[x]$ is an integral domain. HINT: Show that $D[x]$ is commutative, has unity, and contains no zero divisors.

IV.23.2. For $f(x) = x^6 + 3x^5 + 4x^2 + 4x + 2$ and $g(x) = 3x^2 + 2x + 4$ in $\mathbb{Z}_7[x]$, find $q(x)$ and $r(x)$ guaranteed by the Division Algorithm (Theorem 23.1). HINT: In $\mathbb{Z}_7[x]$, all coefficients are from $\{0, 1, 2, 3, 4, 5, 6\}$.

Test 3 #5. Let F be a field and $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \in F[x]$. Prove that if $a \neq 0$ is a zero of f, then a^{-1} is a zero of $g(x) = a_n + a_{n-1}x + \cdots + a_0x^n \in F[x]$.