Introduction to Algebra, MATH 5127

Homework 4, Sections I.6 and I.7

Due Friday September 26, 2014 at 2:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

I.6.27. Find all orders of subgroups of the $\mathbb{Z}_{12}$. Explain why you know that your list is complete.

I.6.50. Let G be a group and suppose $a \in G$ generates a cyclic subgroup of order 2 and it the *unique* such element. Prove that $ax = xa$ for all $x \in G$. HINT: Consider $(xax^{-1})^2$.

I.7.3. List the elements of the subgroup generated by the subset $\{8, 10\}$ of $\mathbb{Z}_{18}$. Give a way to generate all elements of the subgroup and explain why the elements of $\mathbb{Z}_{18}$ which are not in the subgroup (if any) are not in the subgroup.

Test 1, #5. (Due on Tuesday, September 30.) Let G be a group and let $a \in G$. Suppose a has order n. Prove that $\langle a \rangle = \{a, a^2, \ldots, a^{n-1}, a^n = a^0 = e\}$ and $a^i = a^j$ if and only if n divides $i - j$. HINT: Use the Division Algorithm (Theorem 6.3).