Introduction to Algebra, MATH 4127

Homework 4, Sections I.6 and I.7

Due Friday September 26, 2014 at 2:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

I.6.27. Find all orders of subgroups of the $\mathbb{Z}_{12}$. Explain why you know that your list is complete.

I.6.50. Let G be a group and suppose $a \in G$ generates a cyclic subgroup of order 2 and it the *unique* such element. Prove that $ax = xa$ for all $x \in G$. HINT: Consider $(xax^{-1})^2$.

I.7.3. List the elements of the subgroup generated by the subset $\{8, 10\}$ of $\mathbb{Z}_{18}$. Give a way to generate all elements of the subgroup and explain why the elements of $\mathbb{Z}_{18}$ which are not in the subgroup (if any) are not in the subgroup.

Test 1, #5. (Due on Tuesday, September 30.) Prove that $H = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \middle| n \in \mathbb{Z} \right\}$ is a cyclic subgroup of $GL(2, \mathbb{R})$. That is, (1) prove H is a subgroup of $GL(2, \mathbb{R})$ and (2) prove that H is cyclic (by finding a single element of H which generates H and *proving* that this element generates H).