Introduction to Algebra, MATH 5127

Homework 5, Sections II.8 and II.9

Due Friday October 10, 2014 at 2:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

II.8.4. Consider the permutations in S_6:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix} \text{ and } \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}.$$

Calculate $\sigma^{-2}\tau$.

II.8.46. Show that S_n is a nonabelian group for $n \geq 3$. HINT: Compose a rotation with a "flip" (in the examples in class, the "flips" were divided into mirror images and diagonal flips). Do this for general n.

II.8.52. Let G be a group. Consider the functions $\rho_a : G \to G$ where $\rho_a(x) = xa$ for $a \in G$ and $x \in G$.
(a) Prove that each ρ_a is a permutation of set G (that is, it a is one to one and onto function from G to G).
(b) Prove that $P = \{\rho_a \mid a \in G\}$ forms a group under function composition (that is, show closure, associativity, identity, and inverses).
(c) Prove that the group of part (b) is isomorphic to group G.

II.8.A. (Bonus) Let G be a group of permutations on a set S. Let $s \in S$ and define $\text{stab}(s) = \{\alpha \in G \mid \alpha(s) = s\}$. Then $\text{stab}(s)$ is the *stabilizer of s in G*. Prove that $\text{stab}(s)$ is a subgroup of G.

II.9.3. Find all orbits of the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 5 & 1 & 4 & 6 & 8 & 7 \end{pmatrix}.$$

Show your work.

II.9.11 Express

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$$

as a product of disjoint cycles and then as a product of transpositions.