Introduction to Algebra, MATH 5127

Homework 6, Section II.9, Solutions

Due Friday October 17, 2014 at 2:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

II.9.A. The alternating group A_4 consists of all even permutations on the set $\{1,2,3,4\}$. In terms of the cyclic notation and as a product of transpositions, these permutations are: $\alpha_1 = \iota = (1)$, $\alpha_2 = (1,2)(3,4)$, $\alpha_3 = (1,3)(2,4)$, $\alpha_4 = (1,4)(2,3)$, $\alpha_5 = (1,2,3) = (1,3)(1,2)$, $\alpha_6 = (2,4,3) = (2,3)(2,4)$, $\alpha_7 = (1,4,2) = (1,2)(1,4)$, $\alpha_8 = (1,3,4) = (1,4)(1,3)$, $\alpha_9 = (1,3,2) = (1,2)(1,3)$, $\alpha_{10} = (1,4,3) = (1,3)(1,4)$, $\alpha_{11} = (2,3,4) = (2,4)(2,3)$, and $\alpha_{12} = (1,2,4) = (1,4)(1,2)$. Fill in the upper left 6×6 part of this multiplication table for A_4. Show your computations.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}
α_1												
α_2												
α_3												
α_4												
α_5												
α_6												
α_7												
α_8												
α_9												
α_{10}												
α_{11}												
α_{12}												

Does this information reveal any subgroups of A_4?

II.9.27(a). Prove that every permutation in S_n $(n \geq 3)$ can be written as a product of at most $n-1$ transpositions. HINT: On page 90, a cycle is written as a product of transpositions.

II.9.27(b). Prove that every permutation in S_n that is not a cycle can be written as a product of at most $n-2$ transpositions.

II.9.39. Prove that S_n is generated by $\{(1,2),(1,2,\ldots,n)\}$. HINT: Show that as r varies, $\sigma = (1,2,\ldots,n)^r(1,2)(1,2,\ldots,n)^{n-r}$ gives all transpositions in S_n of the form $(1,2)$, $(2,3)$, $\ldots$, $(n-1,n)$, $(n,1)$. Then show that any transposition is a product of transpositions of this special form. NOTE: Since this gives a generating set of S_n of size two, we could use this to produce a Cayley digraph of S_n.