Introduction to Algebra, MATH 4127

Homework 6, Section II.9, Solutions

Due Friday October 17, 2014 at 2:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

II.9.A. The alternating group A_4 consists of all even permutations on the set $\{1, 2, 3, 4\}$. In terms of the cyclic notation and as a product of transpositions, these permutations are: $\alpha_1 = \iota = (1)$, $\alpha_2 = (1, 2)(3, 4)$, $\alpha_3 = (1, 3)(2, 4)$, $\alpha_4 = (1, 4)(2, 3)$, $\alpha_5 = (1, 2, 3) = (1, 3)(1, 2)$, $\alpha_6 = (2, 4, 3) = (2, 3)(2, 4)$, $\alpha_7 = (1, 4, 2) = (1, 2)(1, 4)$, $\alpha_8 = (1, 3, 4) = (1, 4)(1, 3)$, $\alpha_9 = (1, 3, 2) = (1, 2)(1, 3)$, $\alpha_{10} = (1, 4, 3) = (1, 3)(1, 4)$, $\alpha_{11} = (2, 3, 4) = (2, 4)(2, 3)$, and $\alpha_{12} = (1, 2, 4) = (1, 4)(1, 2)$. Fill in the upper left 6×6 part of this multiplication table for A_4. Show your computations.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}
α_1												
α_2												
α_3												
α_4												
α_5												
α_6												
α_7												
α_8												
α_9												
α_{10}												
α_{11}												
α_{12}												

Does this information reveal any subgroups of A_4?

II.9.27(a). Prove that every permutation in S_n $(n \geq 3)$ can be written as a product of at most $n-1$ transpositions. HINT: On page 90, a cycle is written as a product of transpositions.

II.9.27(b). Prove that every permutation in S_n that is not a cycle can be written as a product of at most $n-2$ transpositions.