Introduction to Algebra, MATH 5127

Homework 7, Sections II.10 and II.11

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

II.10.6 & 7. Recall that $H = \{\rho_0, \mu_2\}$ is a subgroup of D_4 (see Table 8.12 on page 80). Find all left cosets and right cosets of H. Are the left cosets the same as the right cosets? Show your computations for each coset.

II.10.40. Prove that if a group G with identity e has finite order n, then $a^n = e$ for all $a \in G$.

II.10.A. Suppose that H and K are subgroups of G and there are elements $a, b \in G$ such that $aH \subseteq bK$. Prove that $H \subseteq K$.

II.10.B. Suppose G is a finite group of order n and let $m \geq 0$ be relatively prime to n. If $g \in G$ and $g^m = e$, prove that $g = e$.

II.11.26. How many abelian groups (up to isomorphism) are there of order 24? of order 25? of order (24)(25)? Give a list of all nonisomorphic groups of each order.

II.11.52. Prove that a finite abelian group G is not cyclic if and only if it contains a subgroup isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$. HINT: You will need Theorem 6.6, 6.10, 6.14, 11.5, and 11.12.