

# Introduction to Algebra, MATH 4127

## Homework 7, Sections II.10 and II.11

**Write in complete sentences!!!** *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

**II.10.6 & 7.** Recall that  $H = \{\rho_0, \mu_2\}$  is a subgroup of  $D_4$  (see Table 8.12 on page 80). Find all left cosets and right cosets of  $H$ . Are the left cosets the same as the right cosets? Show your computations for each coset.

**II.10.40.** Prove that if a group  $G$  with identity  $e$  has finite order  $n$ , then  $a^n = e$  for all  $a \in G$ .

**II.10.A.** Suppose that  $H$  and  $K$  are subgroups of  $G$  and there are elements  $a, b \in G$  such that  $aH \subseteq bK$ . Prove that  $H \subseteq K$ .

**II.11.26.** How many abelian groups (up to isomorphism) are there of order 24? of order 25? of order (24)(25)? Give a list of all nonisomorphic groups of each order.

**II.11.52.** Prove that a finite abelian group  $G$  is not cyclic if and only if it contains a subgroup isomorphic to  $\mathbb{Z}_p \times \mathbb{Z}_p$ . HINT: You will need Theorem 6.6, 6.10, 6.14, 11.5, and 11.12.