Introduction to Algebra, MATH 5127

Homework 8, Sections II.13 and II.14, Solutions

Due Friday October 31, 2014 at 2:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

II.13.A. Let $\phi : \mathbb{R} \to \mathbb{R}^*$, where $\mathbb{R}$ is additive and $\mathbb{R}^*$ (the nonzero real numbers) is multiplicative, be given by $\phi(x) = e^x$. Is ϕ a homomorphism? Is ϕ and isomorphism?

II.13.29. Let G be a group, and let $g \in G$ be fixed. Let $\phi_g : G \to G$ be defined by $\phi_g(x) = gxg^{-1}$ for $x \in G$. Prove that i_g is an isomorphism of G with itself, called an *automorphism* of G.

II.13.B Let $\phi : G \to G'$ and $\psi : G \to G'$ both be group homomorphisms. Let $H = \{g \in G \mid \phi(g) = \psi(g)\}$. Prove or disprove that H is a subgroup of G.

II.13.52. Let $\phi : G \to G'$ be a homomorphism with kernel H and let $a \in G$. Prove the set equality $\{x \in G \mid \phi(x) = \phi(a)\} = Ha$. NOTE: This result relates cosets to kernels of homomorphisms. This will be very important in Section 14. HINT: Let $A = \{x \in G \mid \phi(x) = \phi(a)\}$ and show that $A \subseteq Ha$ and $Ha \subseteq A$.

II.14.7. Find the order of the group $(\mathbb{Z}_2 \times S_3)/(\langle 1, \rho_1 \rangle)$. HINT: The elements of the quotient group are the cosets of $\langle 1, \rho_1 \rangle$.

II.14.34. Prove that if finite group G has exactly one subgroup H of a given order, then H is a normal subgroup of G. HINT: Let $i_g : G \to G$ be the inner automorphism defined as $i_g(x) = gxg^{-1}$ for all $x \in G$. Apply i_g to H and use Exercise 5.41, which implies that $i_g[H]$ is a subgroup of G.