Introduction to Algebra, MATH 5127

Homework 9, Section IV.18, Solutions

Due Friday November 14, 2014 at 2:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. **Justify all steps** by quoting the relevant results from the textbook.

IV.18.11. Consider the set $\{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ with the usual addition and multiplication. Prove that this is a ring. Is every nonzero element a unit (explain)?

IV.18.41. Let p be prime. Prove that in the ring $\mathbb{Z}_p$ we have $(a + b)^p = a^p + b^p$ for all $a, b \in \mathbb{Z}_p$. NOTE: Due to a common misunderstanding of basic algebraic principles, this property is sometimes called *freshman exponentiation*. HINT: You may assume the Binomial Theorem, which holds in $\mathbb{Z}_p$.

18.A. Let $\langle R, +, \cdot \rangle$ be a commutative ring with unity. Suppose a is a unit in R and $b^2 = 0$. Prove that $a + b$ is a unit in R. HINT: Use long division (in your scratch work) to find "$1/(a + b)$" and then show that $a + b$ is a unit with a computation. JUSTIFY ALL STEPS IN YOUR COMPUTATIONS!

18.B. Let $\langle R, +, \cdot \rangle$ be a ring. The *center* of R is the set $S = \{a \in R \mid ar = ra \text{ for all } r \in R\}$. Prove the center of a ring is a subring. HINT: You may use Exercise 18.48 concerning subrings, but justify computations by quoting the use of $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$, or any theorems.