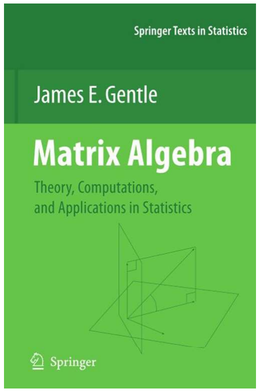
Theory of Matrices

Chapter 2. Vectors and Vector Spaces

2.3. Centered Vectors and Variances and Covariances of Vectors—Proofs of Theorems



Theorem 2.3.1

Theorem 2.3.1. Properties of Covariance.
Let x, y, z be n-vectors and let $a \in \mathbb{R}$. Then:

1. $\text{Cov}(a1_n, y) = 0$,
2. $\text{Cov}(ax, y) = a\text{Cov}(x, y)$,
3. $\text{Cov}(y, y) = V(y)$.

Proof. We use the definition of covariance,
$\text{Cov}(x, y) = \langle x - \overline{x}, y - \overline{y}\rangle/(n-1)$.
1. We have:

$$\begin{aligned}\text{Cov}(a1_n, y) &= \frac{\langle a1_n - a1_n, y - \overline{y}\rangle}{n-1} \text{ since } \overline{a1_n} = a1_n \\ &= \frac{\langle 0, y - \overline{y}\rangle}{n-1} = 0 \text{ by Theorem 2.1.6(1).}\end{aligned}$$

Theorem 2.3.1 (continued)

Proof (continued). We use the definition of covariance,
$\text{Cov}(x, y) = \langle x - \overline{x}, y - \overline{y}\rangle/(n-1)$.

2. Notice that $\overline{ax} = \frac{\sum_{i=1}^n ax_i}{n} = a\frac{\sum_{i=1}^n x_i}{n} = a\overline{x}$, so

$$\begin{aligned}\text{Cov}(ax, y) &= \frac{\langle ax - \overline{ax}, y - \overline{y}\rangle}{n-1} = \frac{\langle ax - a\overline{x}, y - \overline{y}\rangle}{n-1} \\ &= \frac{\langle a(x - \overline{x}), y - \overline{y}\rangle}{n-1} = \frac{a\langle x - \overline{x}, y - \overline{y}\rangle}{n-1} \text{ by Thm 2.1.6(3)} \\ &= a\text{Cov}(x, y).\end{aligned}$$

3. The proof of this is in the class notes before the statement of the theorem. □