# Chapter 1. Basic Vector/Matrix Structure and Notation

**Note.** We will, unless otherwise stated, deal with vectors and matrices with real entries. That is, our scalars will come from the field $\mathbb{R}$. Our terminology will largely overlap with that used in sophomore level Linear Algebra (MATH 2010). Our notation may differ a little, though.

**Definition.** An *n-vector*, where $n \in \mathbb{N}$, is an $n$-tuple of real numbers. The collection of all $n$-vectors make up *n-dimensional real space*, denoted $\mathbb{R}^n$.

**Note.** We may denote $n$-vectors $x$ as either a column vector $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ or as a row vector $x = [x_1, x_2, \ldots, x_n]$. Notice that, unlike in a sophomore level class, we do not use a special symbol to distinguish a vector from a scalar, such as $\vec{x}$ or $\mathbf{x}$; instead we depend on the context to distinguish between vectors and scalars.

**Definition.** A *matrix* is a rectangular array of numbers. An $n \times m$ matrix is a matrix with $n$ rows and $m$ columns:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} = [a_{ij}].$$

The set of all $n \times m$ matrices with real entries is denoted $\mathbb{R}^{n \times m}$.

**Note.** In this course, all vectors and matrices are "finite." The study of infinite vectors and matrices belongs to the realm of analysis (in particular, we cover the topics in our Fundamentals of Functional Analysis [MATH 5740]).

**Note.** On pages 7 and 8 the text describes several ways that matrices and vectors can by used to represent data.

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