Chapter 5. Matrix Transformations and Factorizations

Note. We encountered a number of matrix factorizations (or "matrix decompositions") in Chapter 3. In this chapter we introduce three new factorizations:

(1) the LU (and LR and LDU) factorization of a general matrix (in Section 5.6),

(2) the QR factorization of a general matrix (in Section 5.7), and

(3) the Cholesky factorization of a nonnegative definite matrix (in Section 5.9).

Section 5.1. Transformations by Orthogonal Matrices

Note. In Chapter 3 we saw for orthogonal Q (that is, $Q^TQ = I$) that $\|Qx\|_2 = \|x\|_2$, so orthogonal transformations preserve vector lengths.

Note. If Q is orthogonal then for vectors x and y

$$\langle Qx, Qy\rangle = (Qx)^T(Qy) = x^TQ^TQy = x^Ty = \langle x, y\rangle.$$

We define the angle between vectors x and y as $\arccos\left(\frac{\langle x, y\rangle}{\|x\|_2\|y\|_2}\right)$, so

$$\arccos\left(\frac{\langle Qx, Qy\rangle}{\|Qx\|_2\|Qy\|_2}\right) = \arccos\left(\frac{\langle x, y\rangle}{\|x\|_2\|y\|_2}\right).$$

So an orthogonal transformation preserves angles between vectors. (In analysis, this called a "conformal" mapping.)

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