Section 5.8. Singular Value Factorization

Note. Gentle declares the singular value decomposition "useful in solving linear systems." We recall the definition and existence of singular value decompositions from Section 3.8. Remember that the words "factorization" and "decomposition" are being used interchangeably.

Definition. For an $n \times m$ matrix A, a factorization $A = UDV^T$, where U is an $n \times n$ orthogonal matrix, V is an $m \times m$ orthogonal matrix, and D is an $n \times m$ diagonal matrix with nonnegative entries is a *singular value decomposition* of A. (An $n \times m$ diagonal matrix has $\min\{n, m\}$ elements on the diagonal and all other entries are zero.) The nonzero entries of D are the *singular values* of A.

Theorem 3.8.16. Let A be an $n \times m$ matrix. Then there exists a singular value decomposition of A.

Note. Gentle declares the singular value decomposition as "rank revealing" since the rank of A is the number of nonzero singular values as given by the diagonal of D (see Theorem 3.3.3; U and V are square orthogonal matrices and so are invertible and are products of elementary matrices).

Revised: 2/7/2018