Real Analysis 1, MATH 5210, Fall 2022

Homework 10, Section 3.2 Sequential Pointwise Limits and Simple Approximation

Due Saturday, November 5, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

3.12. Let f be a bounded measurable function on E. Prove there are sequences of simple functions on E, $\{\varphi_n\}$ and $\{\psi_n\}$, such that $\{\varphi_n\}$ is increasing and $\{\psi_n\}$ is decreasing and each of these sequences converges to f uniformly. HINT: Use partitions and refinements of these partitions.

3.14. Let f be a measurable function on E that is finite a.e. on E and $m(E) < \infty$. Prove that for each $\varepsilon > 0$, there is a measurable set F contained in E such that f is bounded on F and $m(E \setminus F) < \varepsilon$.

3.15. Let f be a measurable function on E that is finite a.e. on E and $m(E) < \infty$. Prove that for each $\varepsilon > 0$ there is a measurable set F contained in E and a sequence $\{\varphi_n\}$ of simple functions on E such that $\{\varphi_n\} \to f$ uniformly on F and $m(E \setminus F) < \varepsilon$. HINT: Use Exercises 3.12 and 3.14.

3.16. (Bonus) Let I be a closed, bounded interval and E a measurable subset of I. Let $\varepsilon > 0$. Prove that there is a step function h on I and a measurable subset F of I for which

$$h = \chi_E \text{ on } F \text{ and } m(I \setminus F) < \varepsilon.$$

HINT: Use Theorem 2.12 as applied to set E and find $\{I_k\}_{k=1}^n$. Then define $I_k' = I_k \cap I$ so that $I_k' \subset I$. Define F and h. NOTE: This result shows that a characteristic function on a bounded measurable set is "nearly" a step function.