

Real Analysis 1, MATH 5210, Fall 2022

Homework 11, Section 4.2 The Lebesgue Integral of a Bounded Measurable Function on a Set of Finite Measure

Due Saturday, November 12, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

4.9. Let E have measure zero. If f is bounded on E then f is measurable and $\int_E f = 0$.

4.11. Prove by example that the Bounded Convergence Theorem does not hold for Riemann integrals. HINT: Create sequence $\{f_n\}_{n=1}^\infty$ of Riemann integrable functions such that $f = \lim_{n \rightarrow \infty} f_n$ is not Riemann integrable.

4.16. Let f be a nonnegative bounded measurable function on a set E of finite measure. If $\int_E f = 0$ then $f = 0$ a.e. on E .

4.14. (Bonus) Prove that Proposition 4.8 is a special case of the Bounded Convergence Theorem. HINT: Let $|f_n| \leq M_n$ on E and $|f| \leq M_\infty$ on E . Find M as described in the Bounded Convergence Theorem.